Uranian Satellite Formation

- Satellite formation from a debris disk produced by a giant impact
  Ishizawa, Sasaki & Hosono, ApJ 885, 132 (2019)

- Satellite formation via vapor disk evolution
  Ida, Ueta, Sasaki & Ishizawa, Nature Astronomy, 4, 880 (2020)

- Satellite formation from a debris disk after disk evolution
  Kihara, Sasaki & Ida, submitted

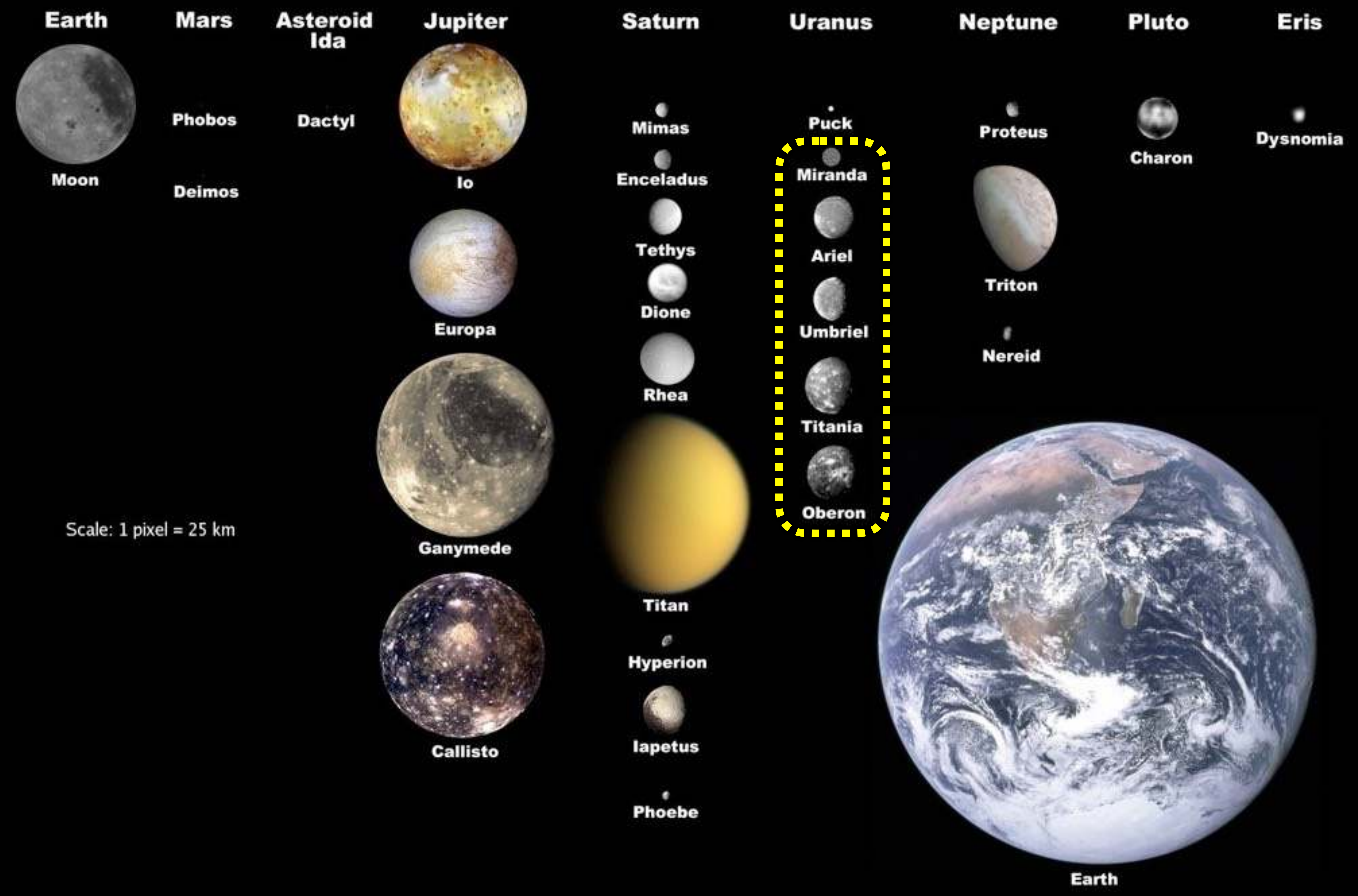
- Giant impact of Uranus to produce the disk matereial
  Murashima & Sasaki, in prep.

Takanori Sasaki

Tuesday Seminar 2025/04/08

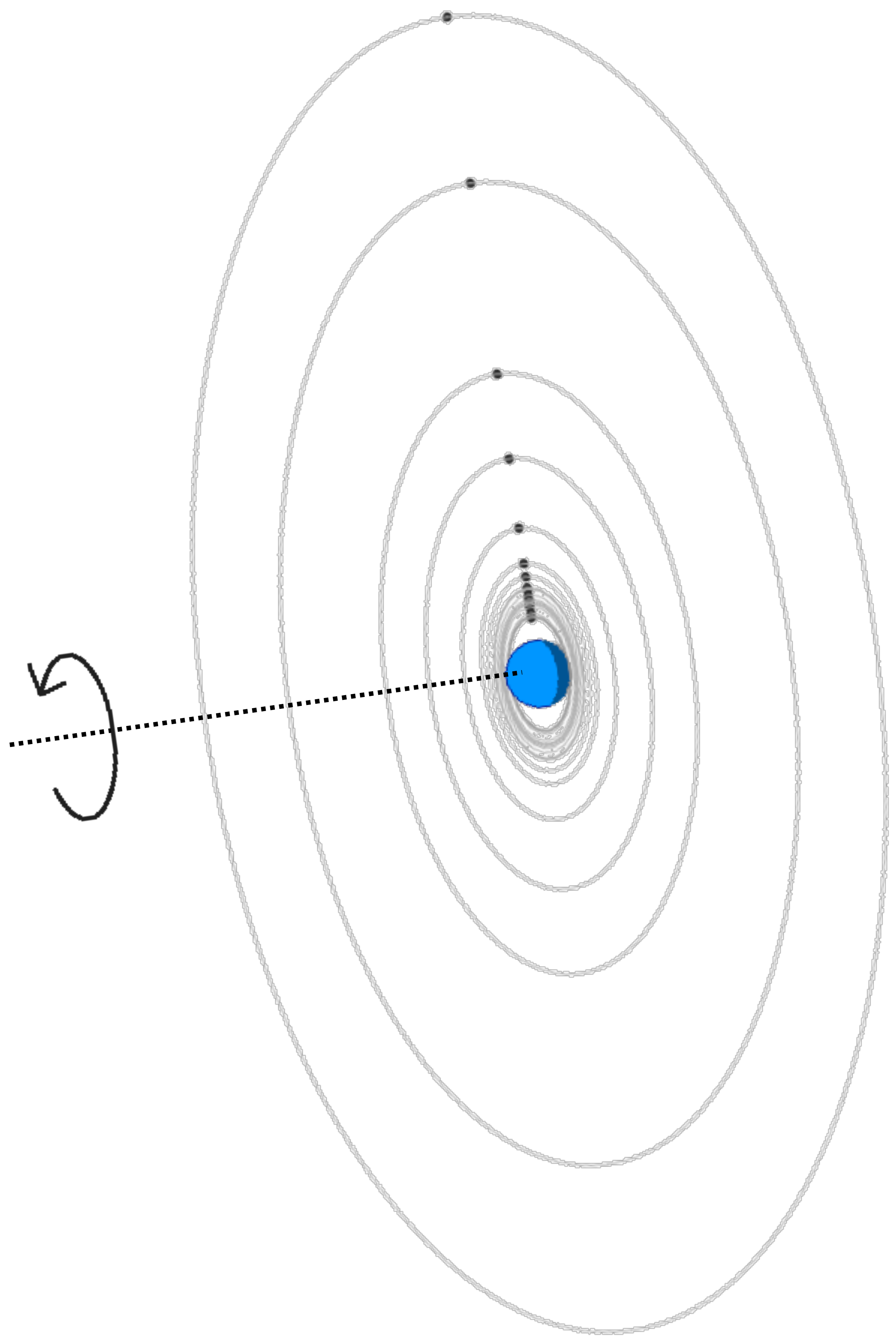
Introduction

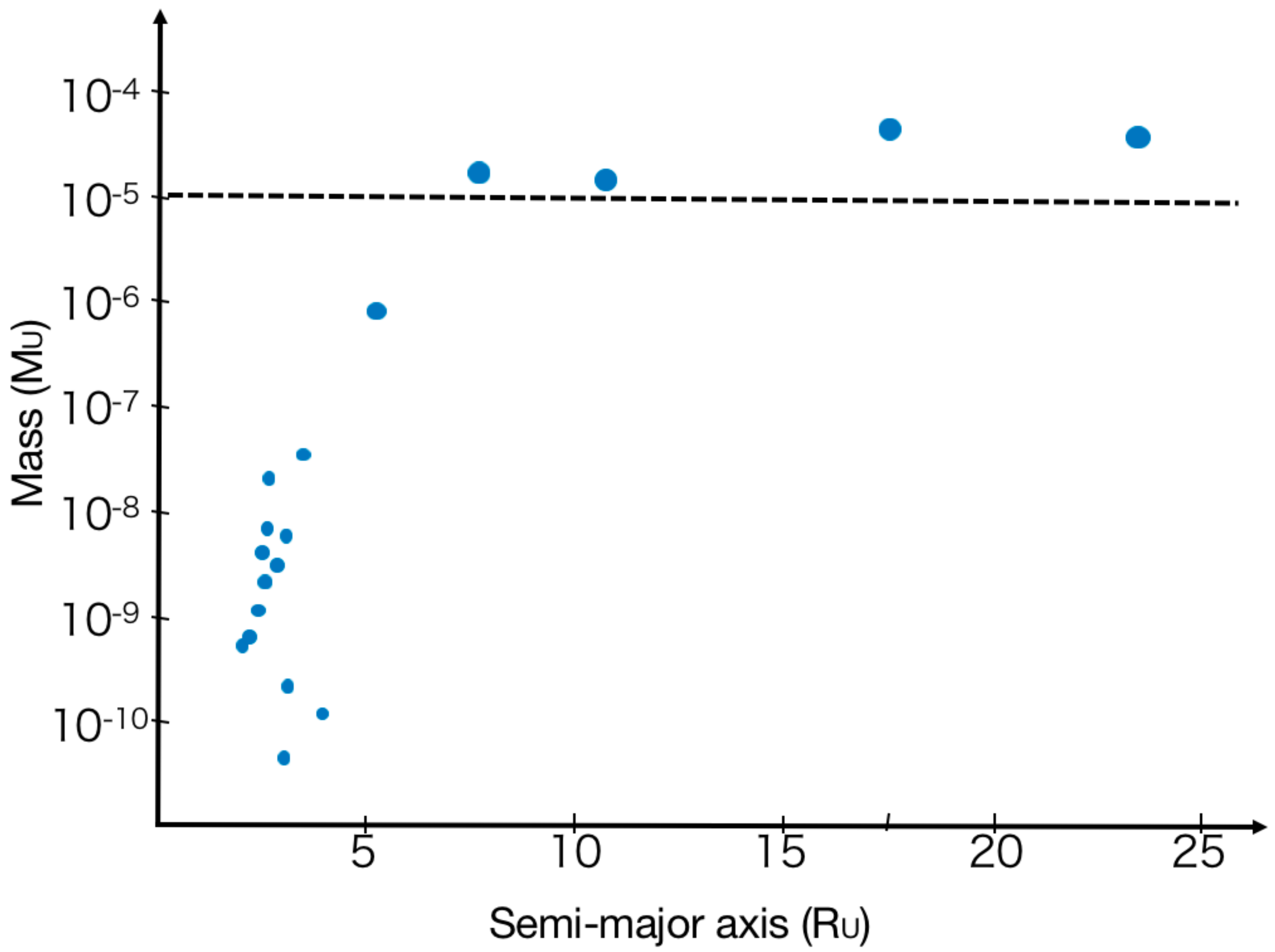
Selected Moons of the Solar System, with Earth for Scale



Uranian satellite system

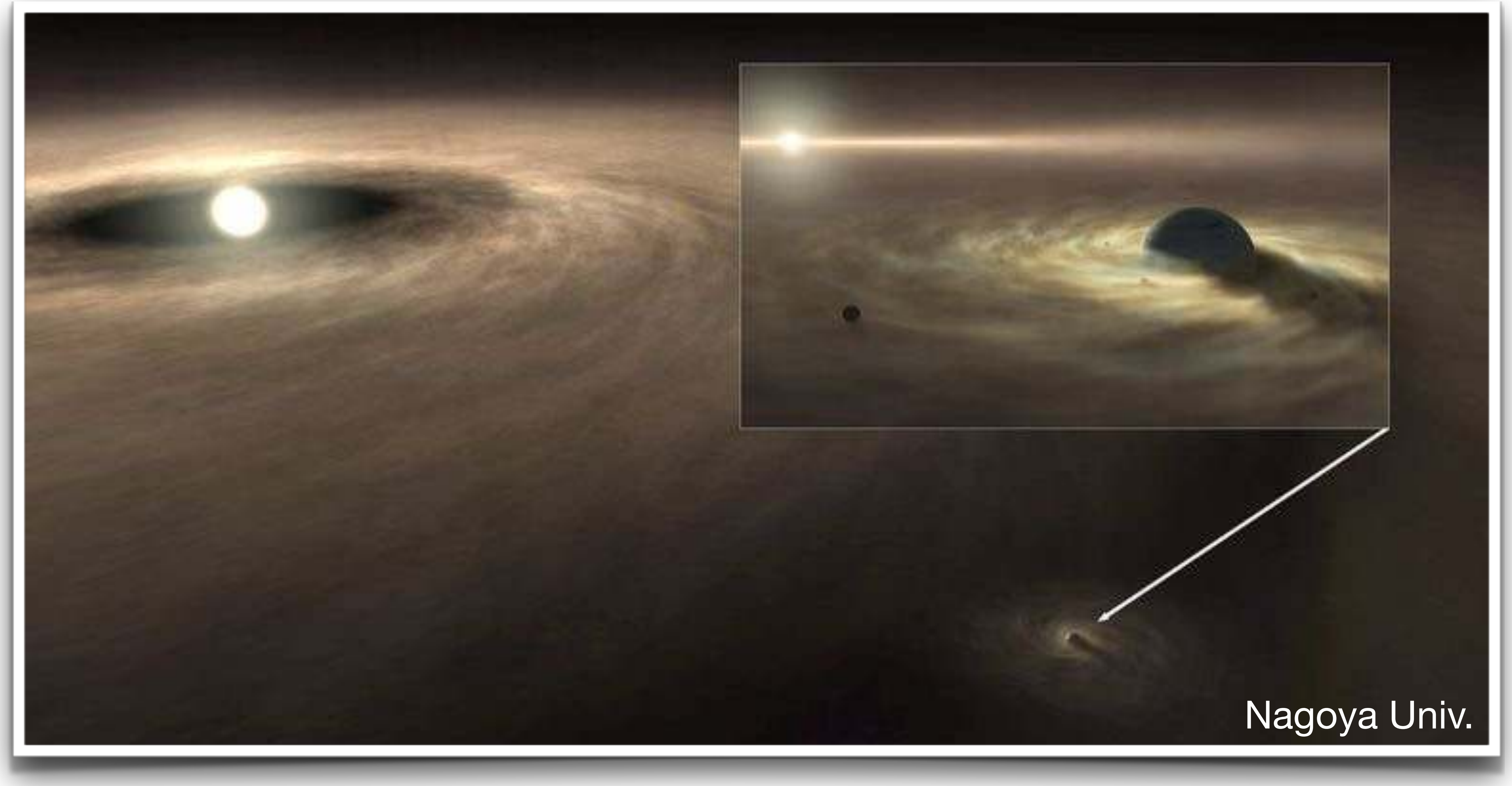
Line up on Uranus' tilted equatorial plane





“Gas-starved” circum-planetary disk model

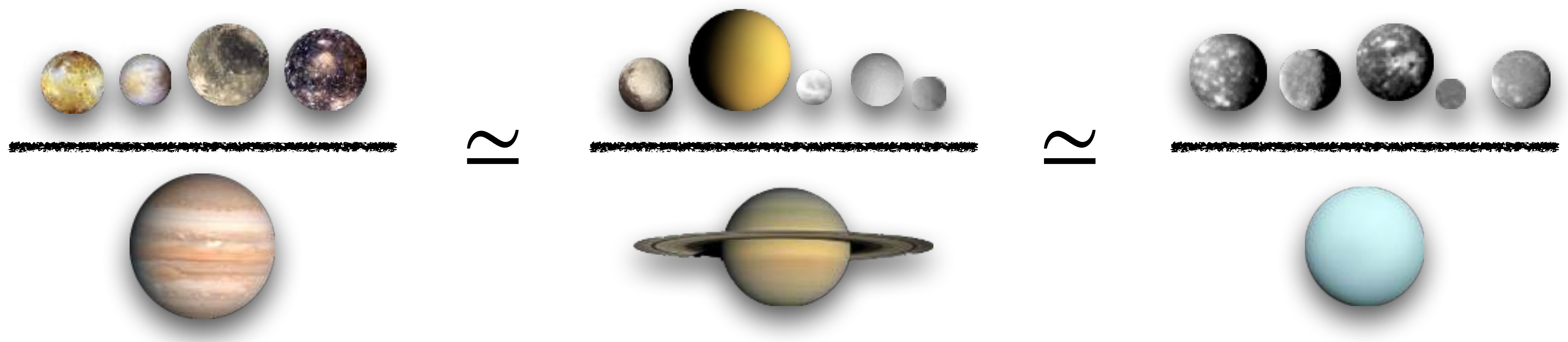
[Canup & Ward, *ApJ*, 2002; *Nature*, 2006]



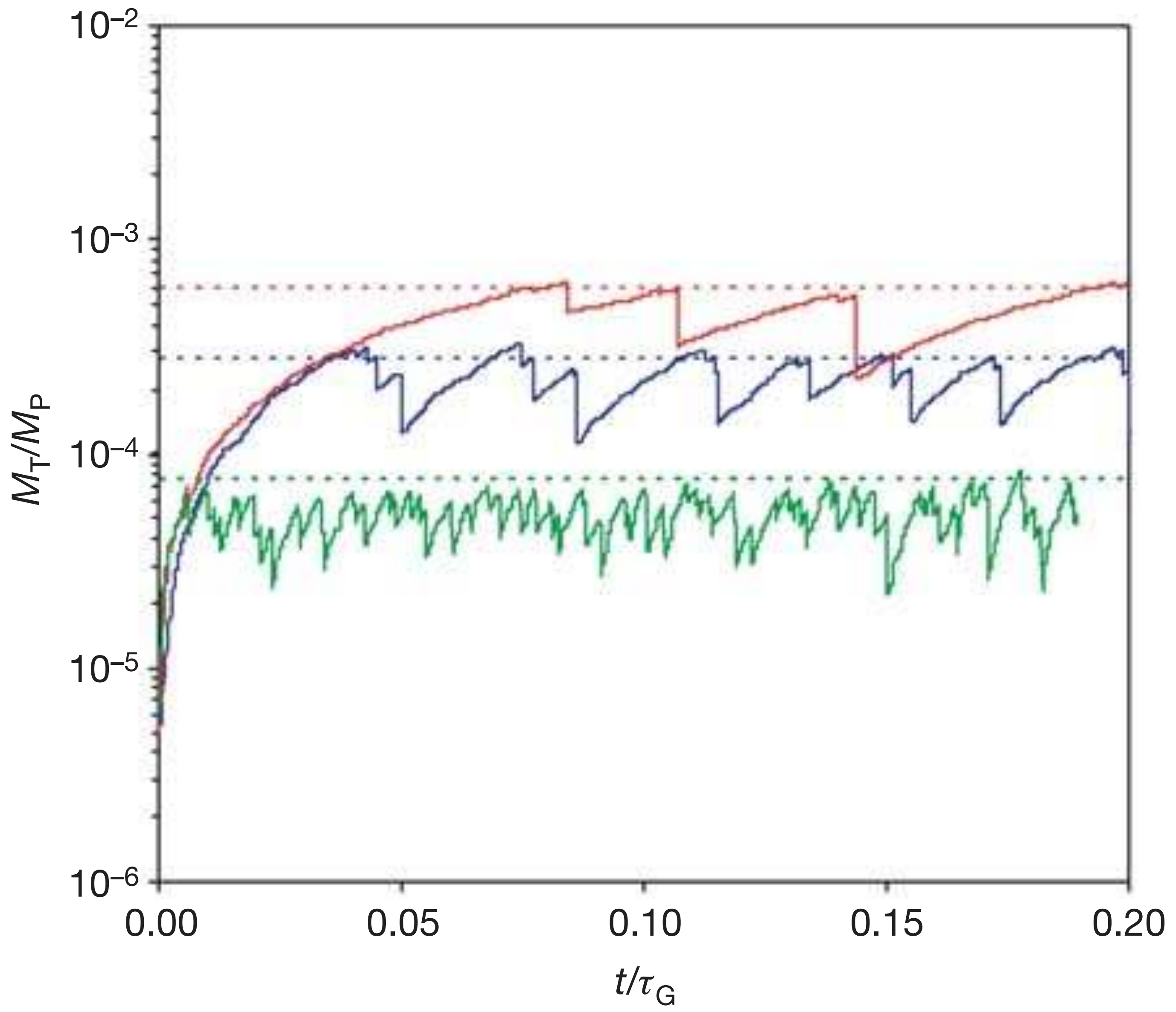
An accretion disk produced by a slow inflow of gas and solids is consistent with conditions needed to form Jovian/Saturnian satellites.

Common mass scaling for satellite systems

Why so similar?



$\sim 10^{-4}$



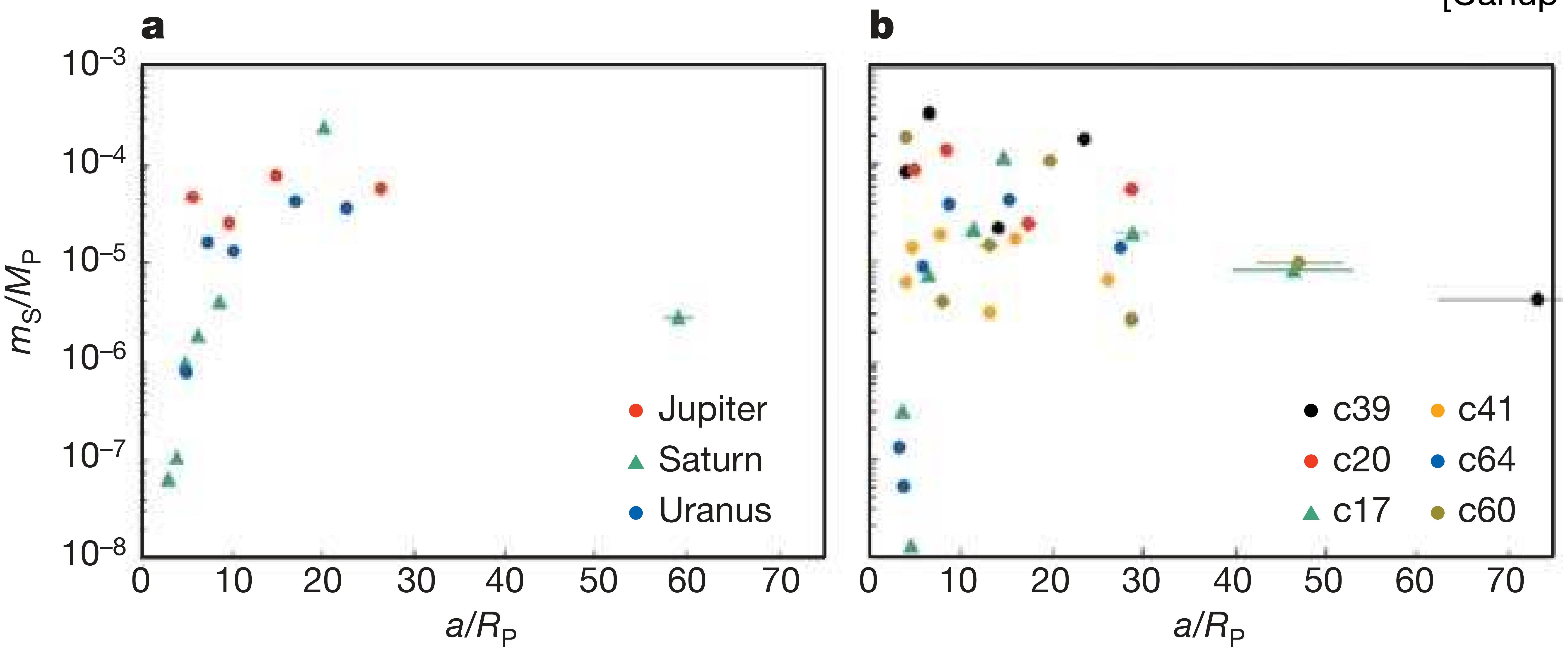
The mass fraction is regulated to ~10^{-4} by a balance between the supply of inflowing material and satellite loss through orbital decay.

[Canup & Ward, *Nature*, 2006]

Circum-planetary disk scenario

Shoot often, hit at last

[Canup & Ward, *Nature*, 2006]

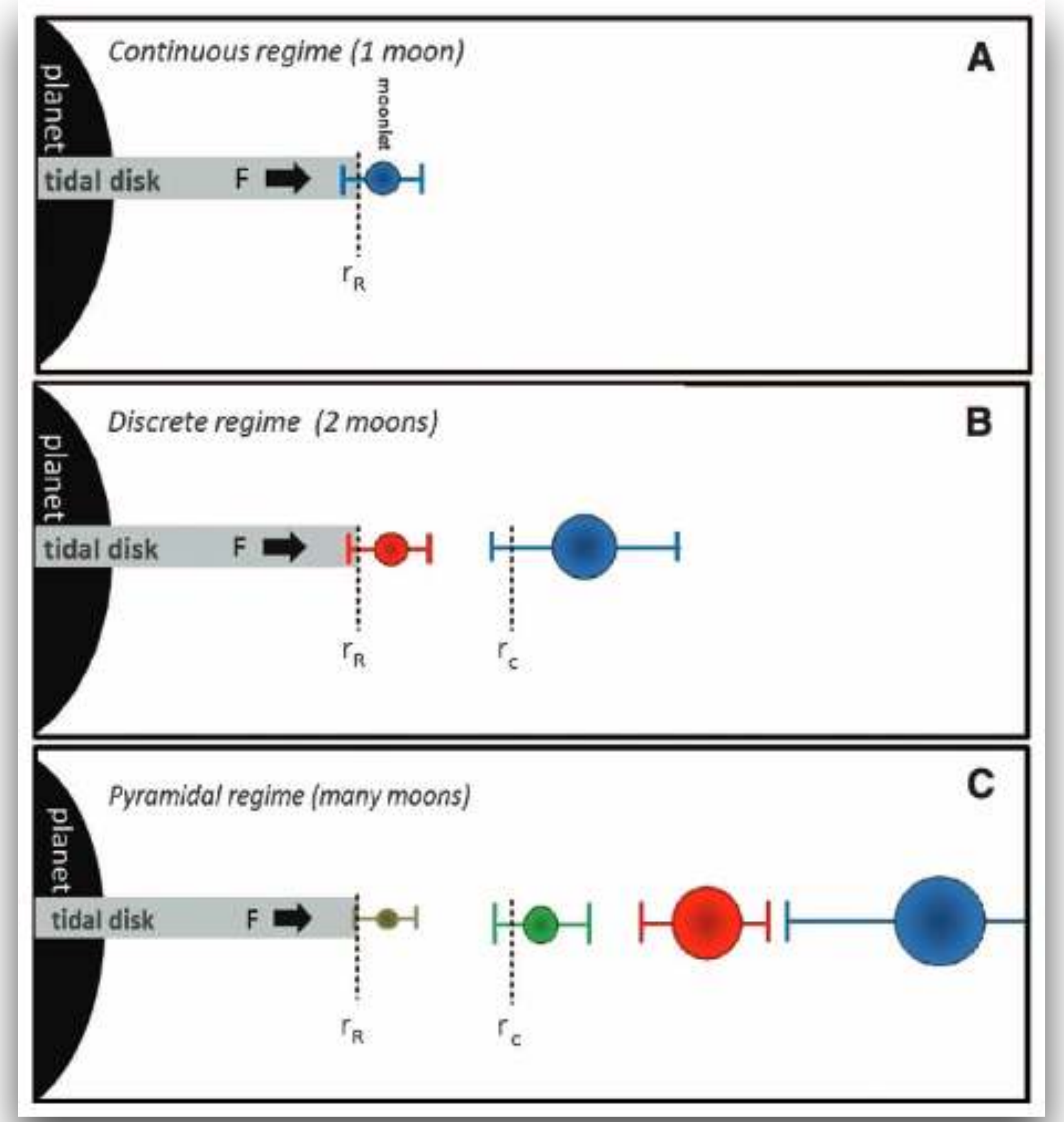


Satellites' properties are consistent with some of the simulation results, but the 98° tilt of Uranus's rotational axis requires additional explanation.

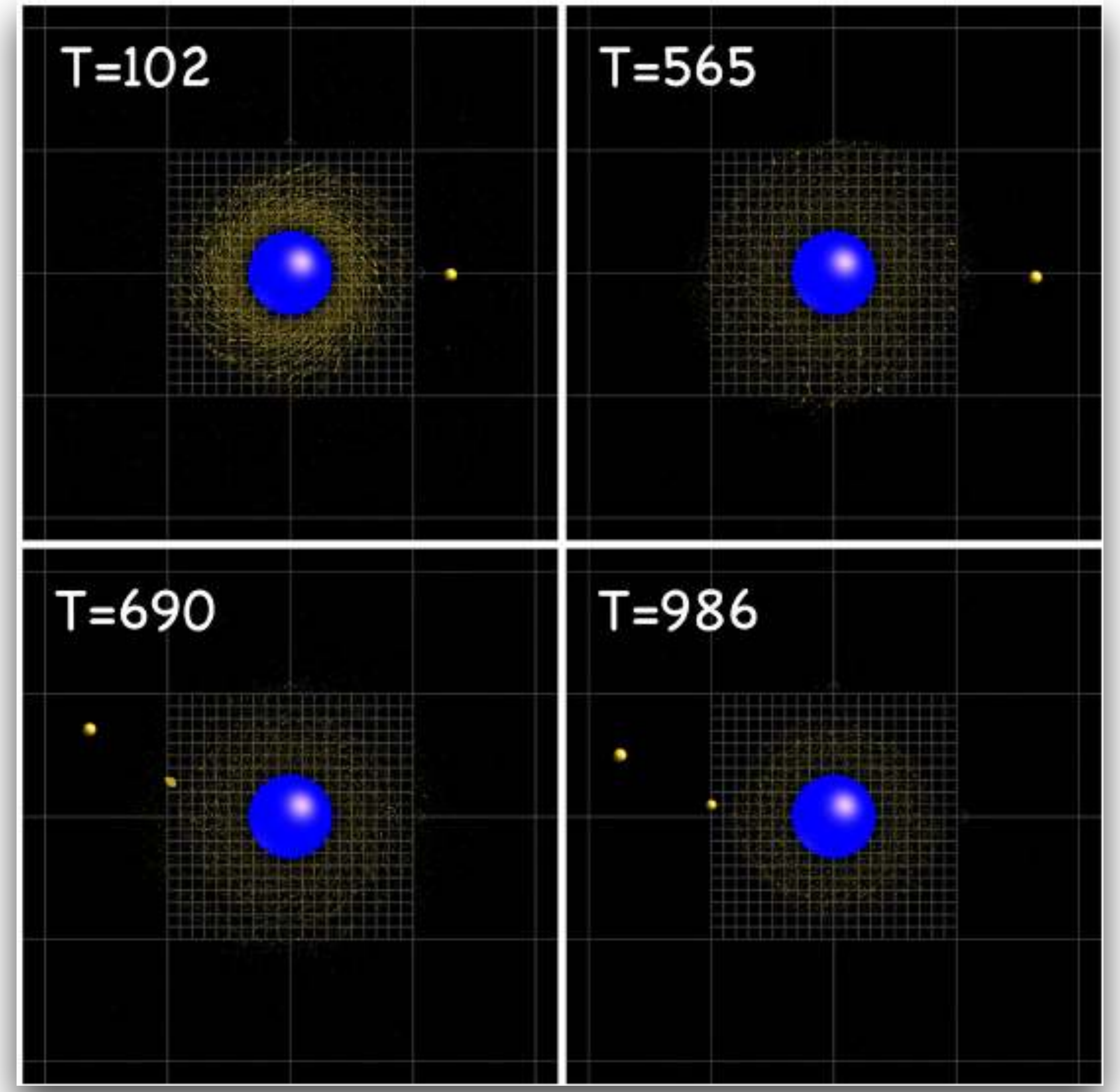
Satellite formation via tidal disk

The earlier, the bigger

A retinue of satellites appear with masses increasing with distance to the planet.



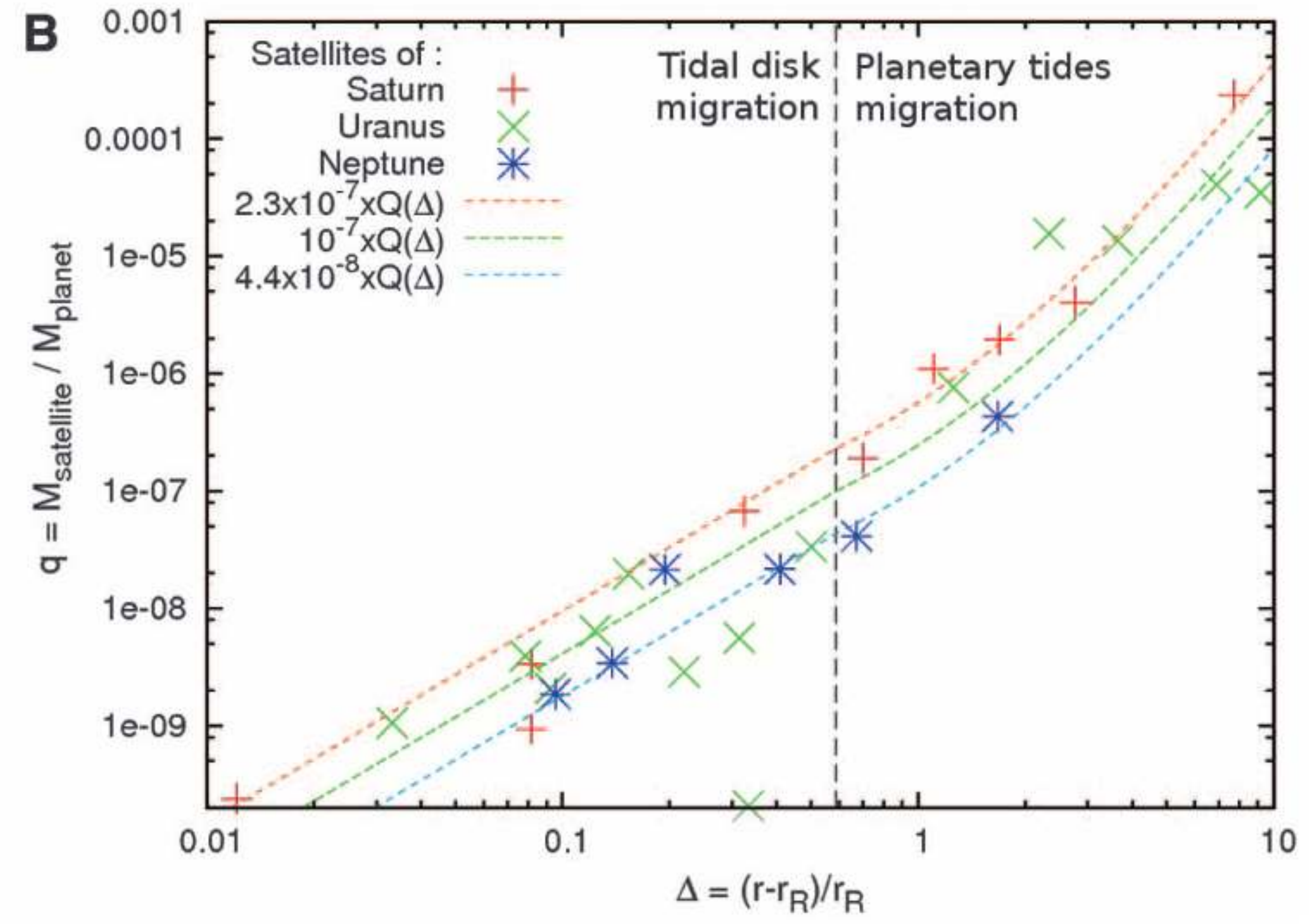
[Crida & Charnoz, *Science*, 2012]



[Hyodo & Otsuki, *ApJ*, 2015]

Tidal disk scenario

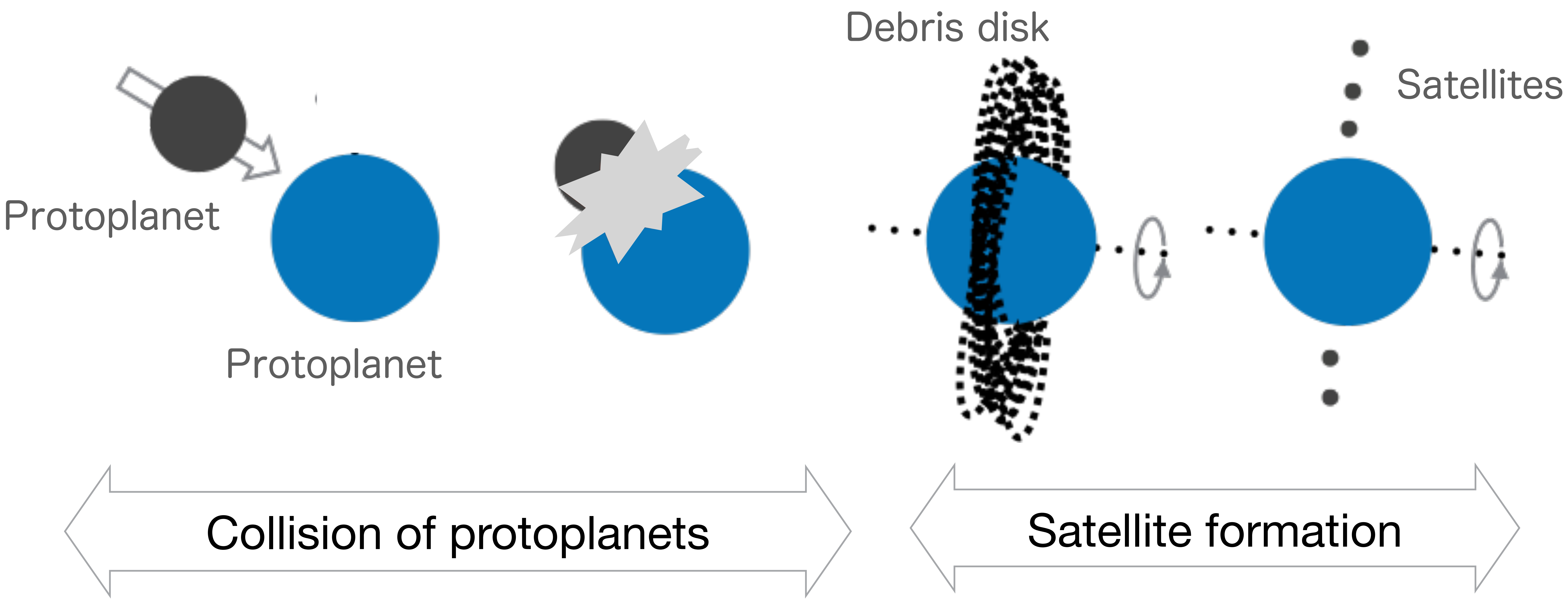
All giants wore massive rings



[Crida & Charnoz, *Science*, 2012]

Satellites' properties are consistent (?) with the simulation, but they assume unrealistic strong dissipation and requires tilting scenario.

Giant Impact (GI) scenario for satellite formation



GI scenario can explain

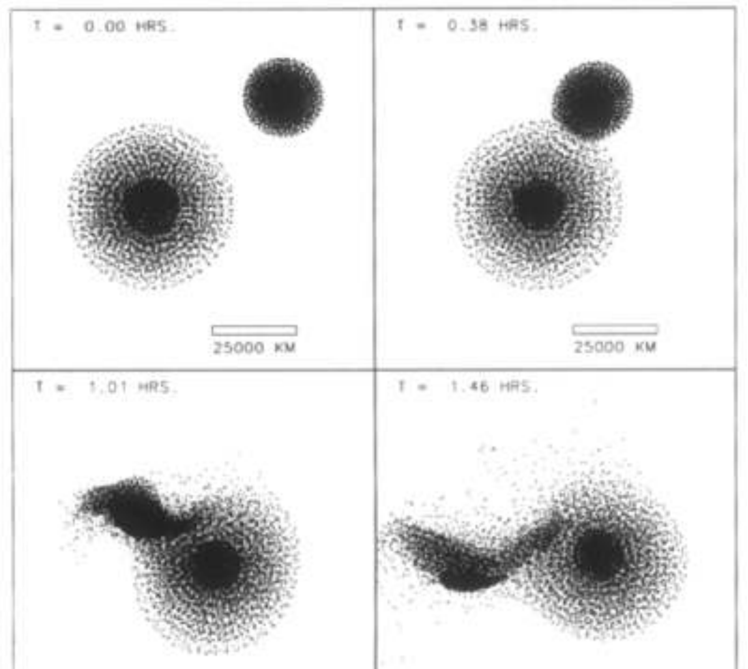
- **Large axial tilt of Uranus ~98°**
- **Regular satellites formation along the equatorial plane**

simultaneously

GI scenario for Uranian system

<u>Giant impact simulations</u> for Uranus

using SPH (Smoothed Particle Hydrodynamics method)



Slattery et al. (1992)

Slattery et al. (1992) suggest that

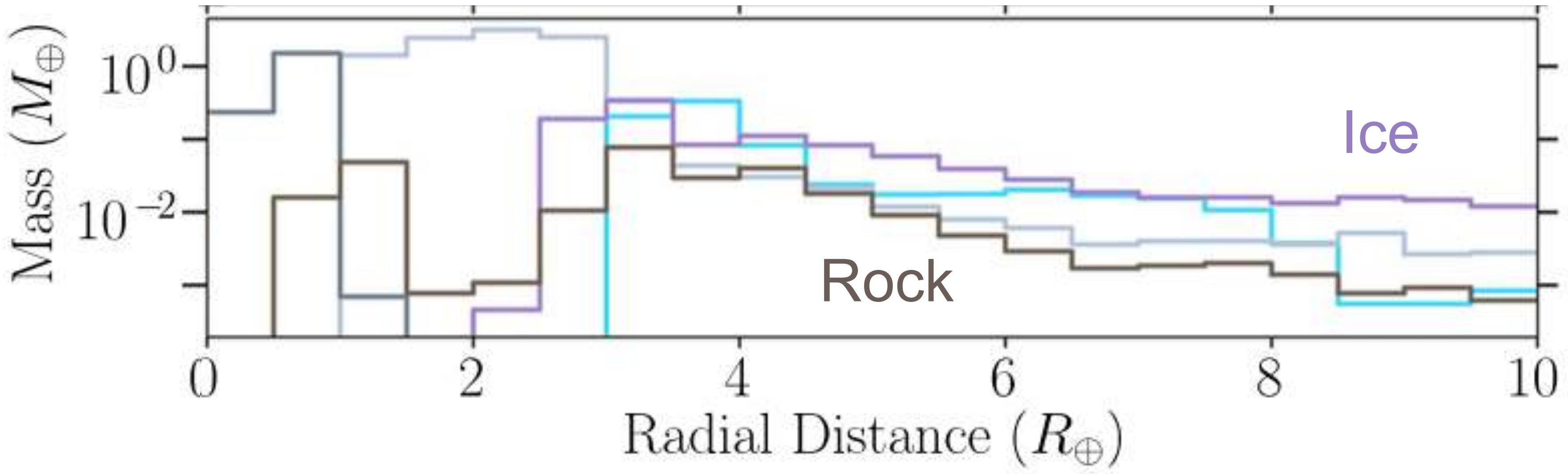
- a GI could produce the current spin period of Uranus
- **ejected materials could have sufficient mass for the formation of Uranian satellites**

But the details of the circumplanetary disk is still unknown due to low resolution of their simulations ($N_{Uranus+Impactor}$=8,000)

➡ **Keggereis et al. (2018) performed similar simulations in high resolution (N≦100,000)**

Kegerreis et al. (2018) suggest that

- ejected materials could have sufficient mass of ice and rock for the formation of Uranian satellites
- **the surface density of the disk has a negative power-law distribution**

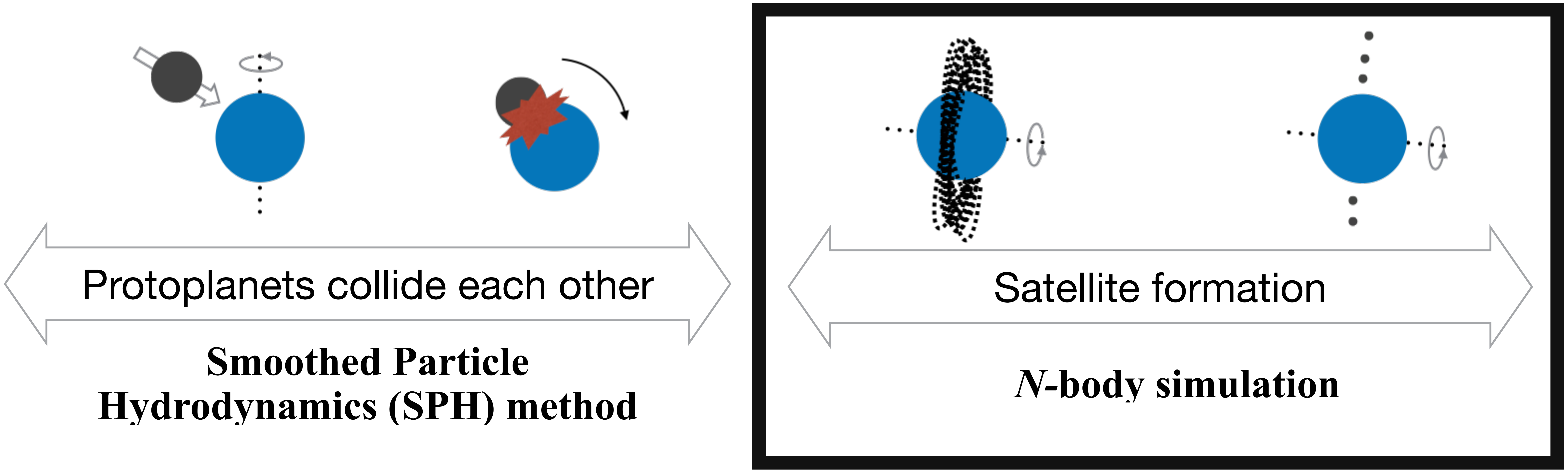


Kegerreis et al. (2018)

But whether the current satellite can form from such a disk had not been investigated in accumulation simulation

Ishizawa et al. (2019)

Ishizawa, Sasaki & Hosono (2019)



To investigate whether the current satellites can form through GI scenario or not by performing *N*-body simulation

Method (N-body simulation)

- Equation of motion : $$\frac{d^2\boldsymbol{r_i}}{dt^2} = -\sum_{j\neq i} Gm_j \frac{\boldsymbol{r_i}-\boldsymbol{r_j}}{|\boldsymbol{r_i}-\boldsymbol{r_j}|^3} \quad (i, j = 1, 2, ..., N)$$

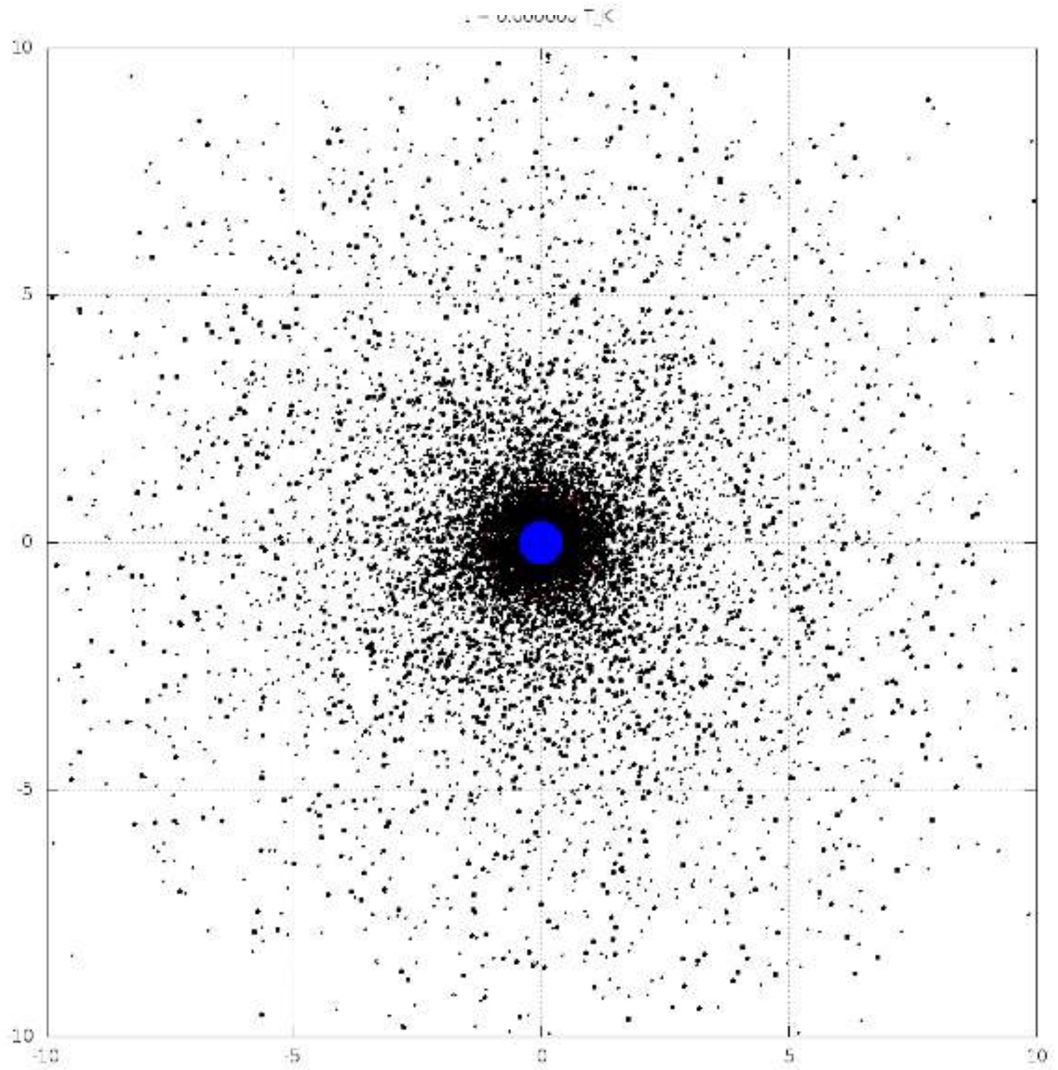
- **Number of disk particles : N_{disk} = 10,000**

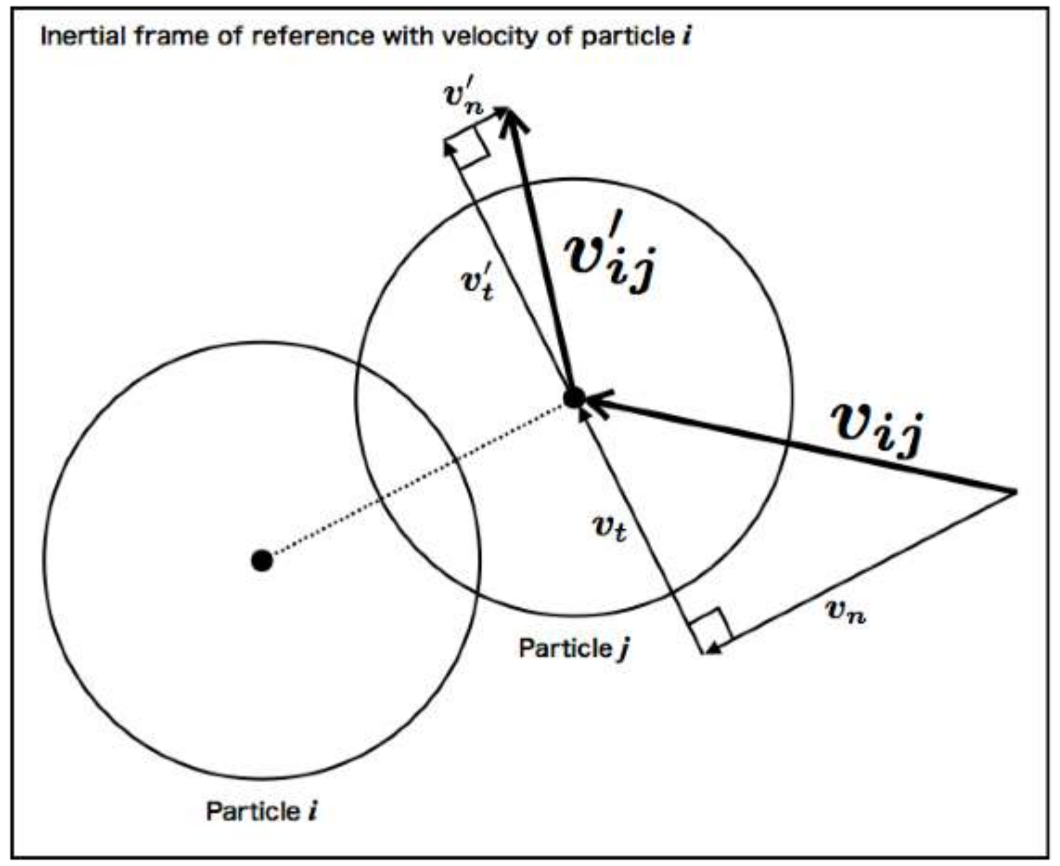
- **Collisions** are moderately inelastic, and **mergers** occur if the Jacobi energy after the collision is negative (e.g., Kokubo et al., 2000)

- Using 4th Hermite scheme and Leap Frog method for the numerical time integration

- Using FDPS for speeding up calculations

(Framework for Developing Particle Simulator ; Iwasawa et al., 2016)

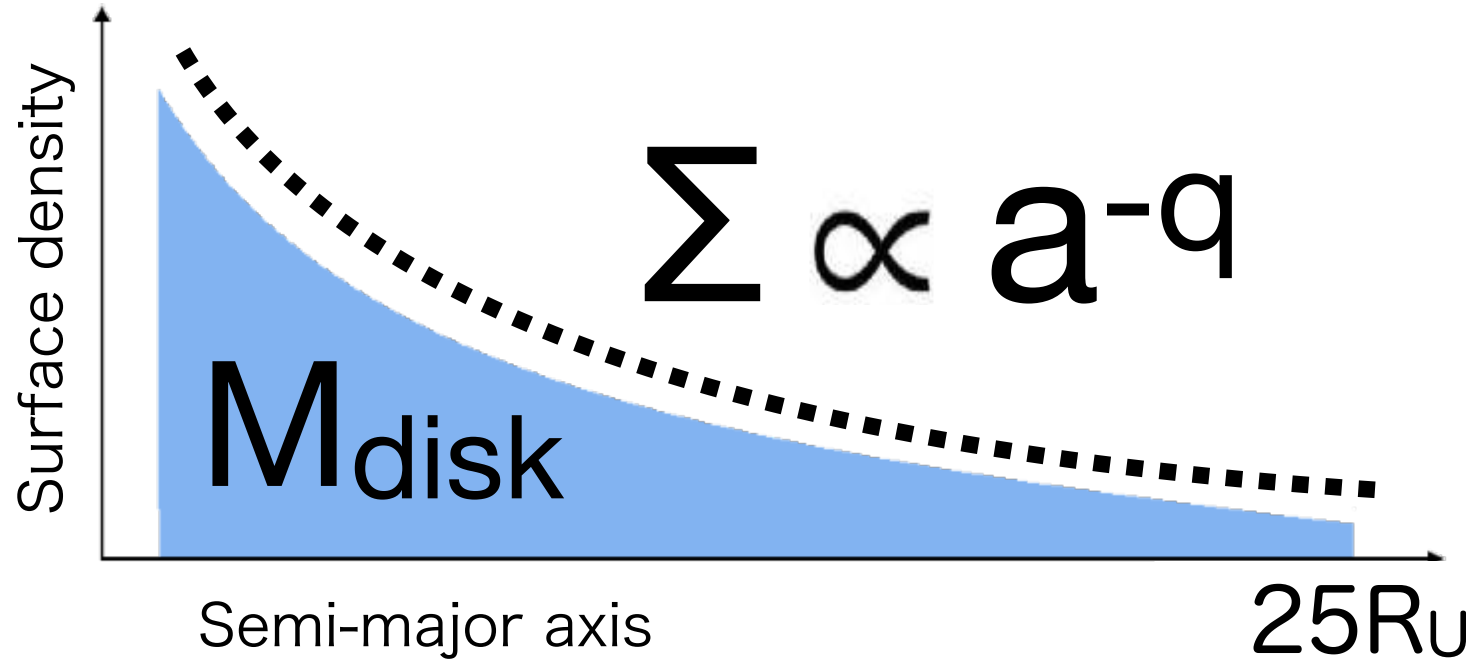




Disk models with negative gradient

Surface density of a circumplanetary disk generated by a GI is assumed to have a power-low distribution

- Disk size a → $R_U < a < 25R_U$
- Disk mass M_{disk} → Several times M_{tot} (the total satellite mass)
- Surface density $\Sigma(a) \propto a^{-q}$ → The power-index -*q* is varied as a parameter



	Disk1	Disk2	Disk3	Disk4	Disk5	Disk6
M_{disk}	$4M_{tot}$	$3M_{tot}$	$3M_{tot}$	$4M_{tot}$	$3M_{tot}$	$10M_{tot}$
q	2.15	1.50	1.95	1.95	3.0	2.15

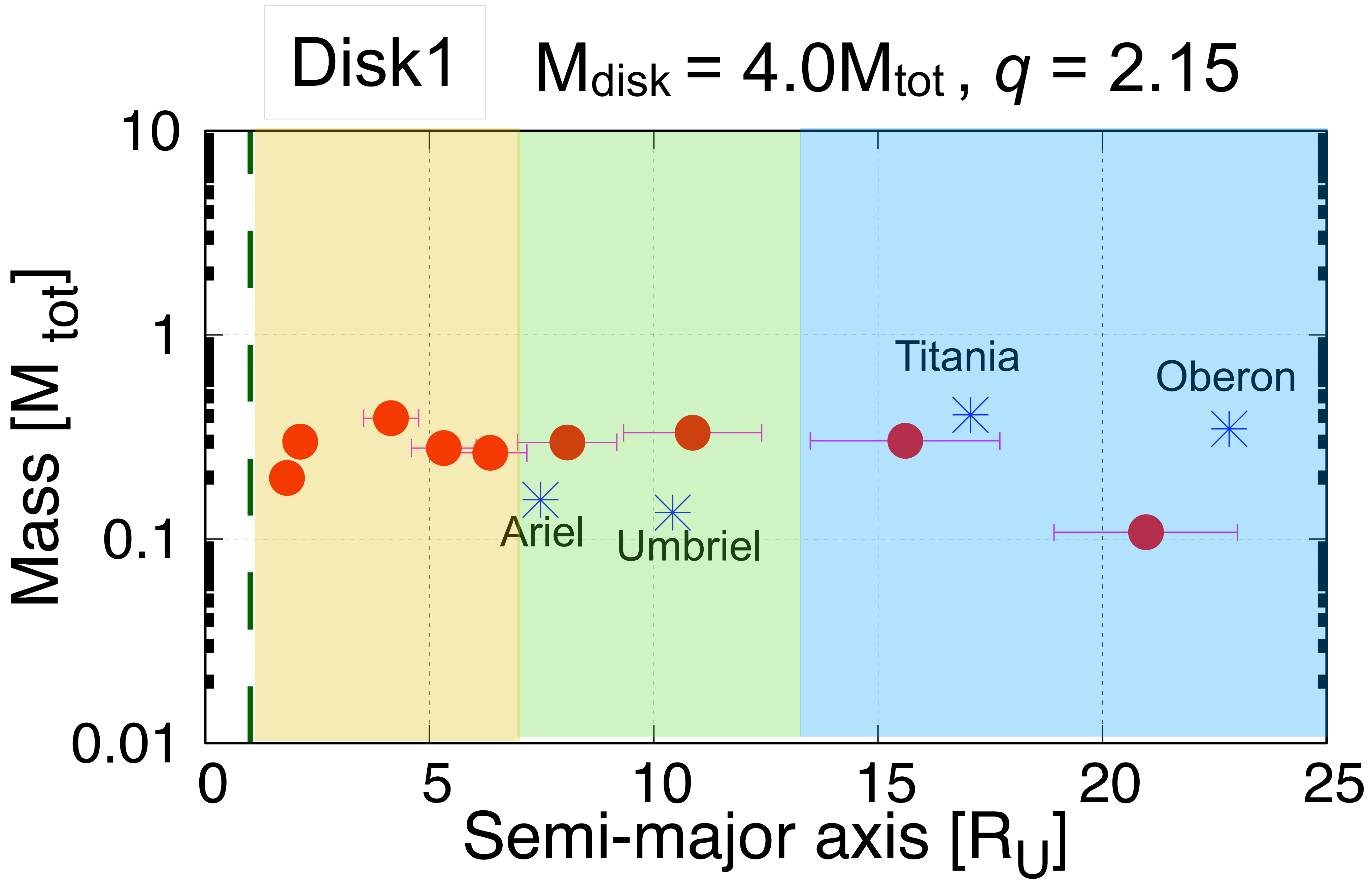
These disk models have **negative gradients** directly inferred from the results of the SPH simulations (Kegerreis et al., 2018)

Results of N-body simulations

for a disk model with negative gradient

Results of Numerical simulations ●

The current satellites ✳



$R_U < a < 7R_U$	extra massive satellites
$7R_U < a < 13R_U$	too large satellites
$13R_U < a$	too small satellites

After several thousand years,

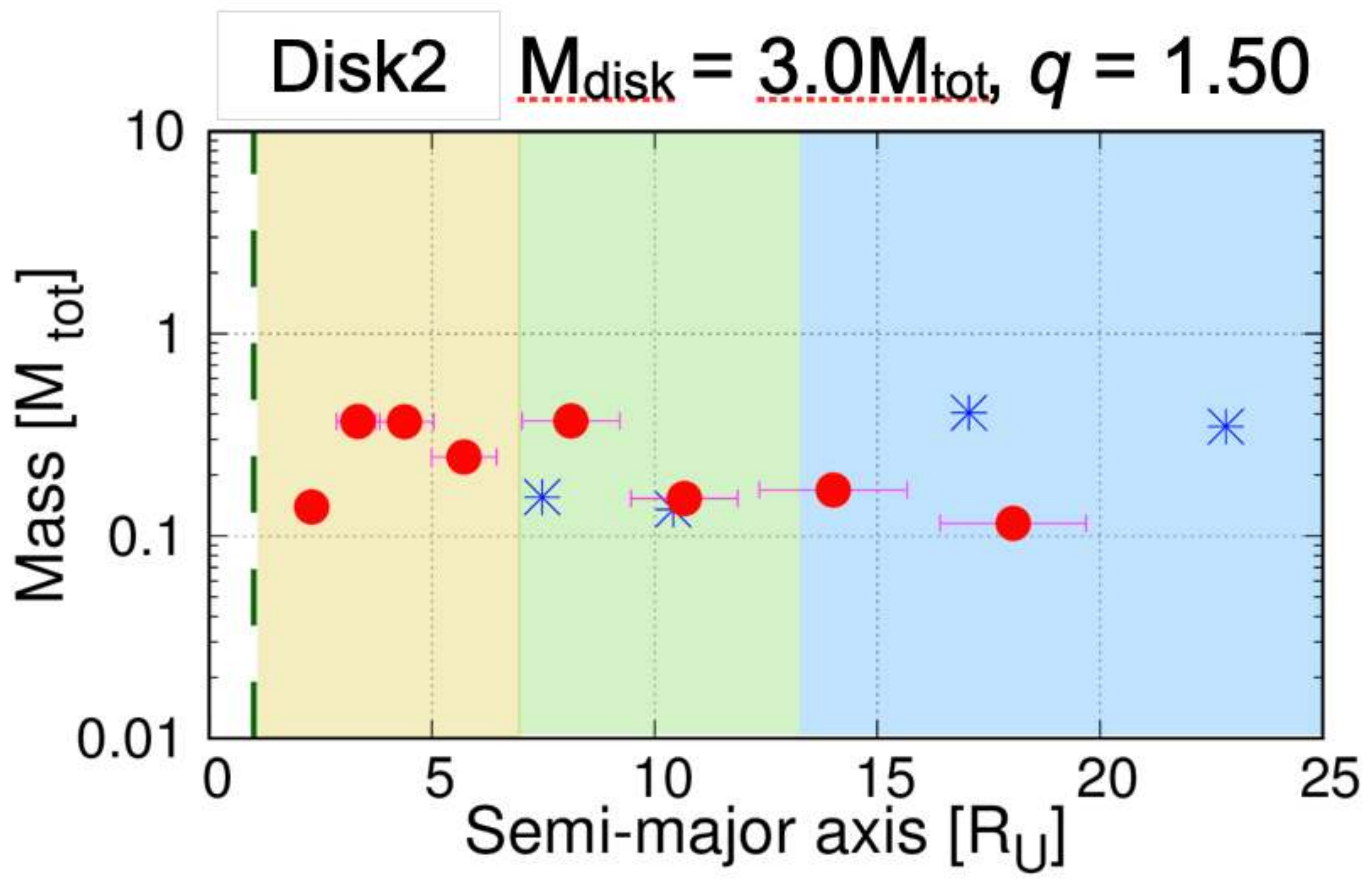
- **some particles have comparable masses to the current satellites**
- **but the mass-orbital distribution is different from the current system**

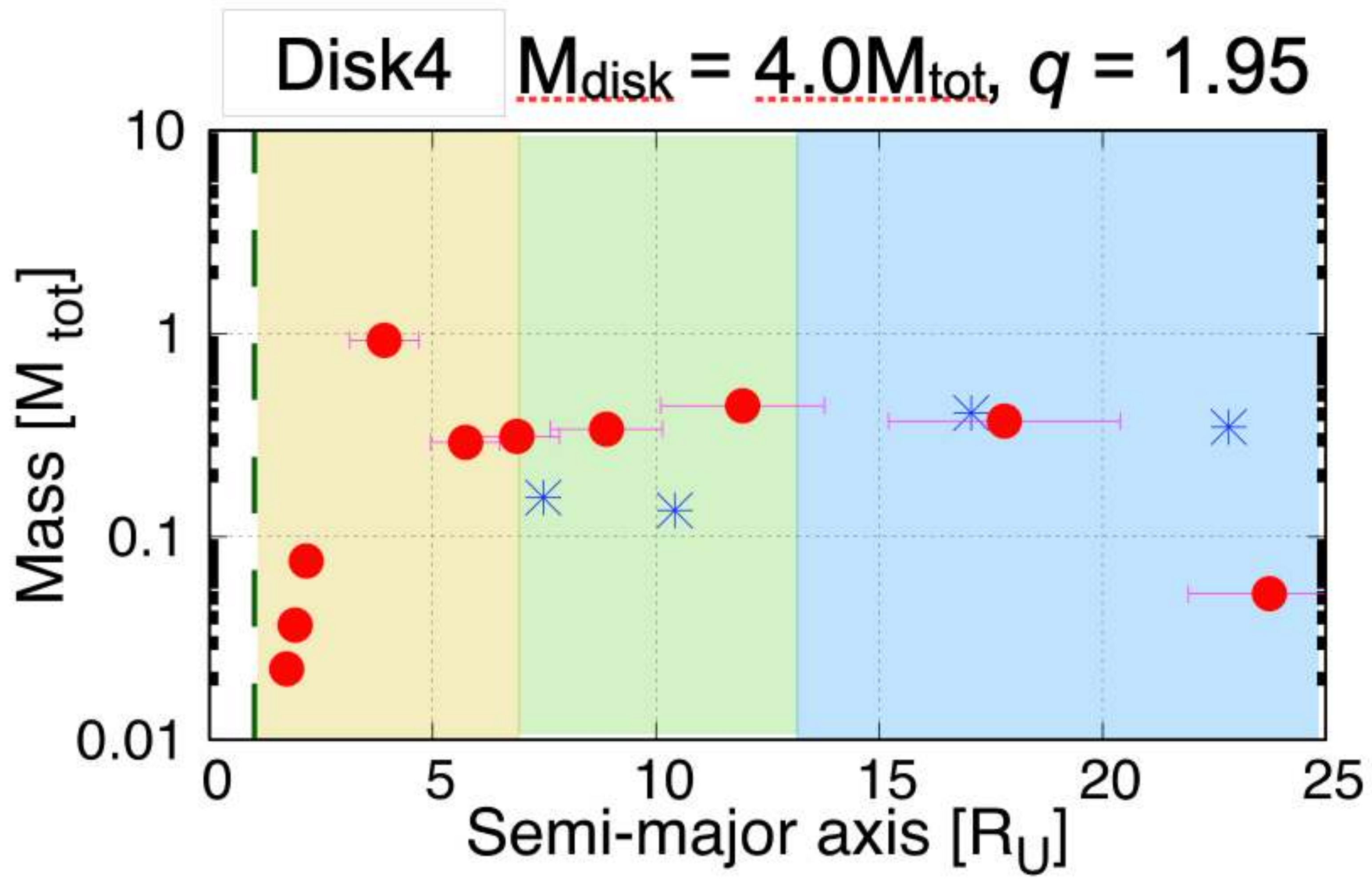
Results of N-body simulations

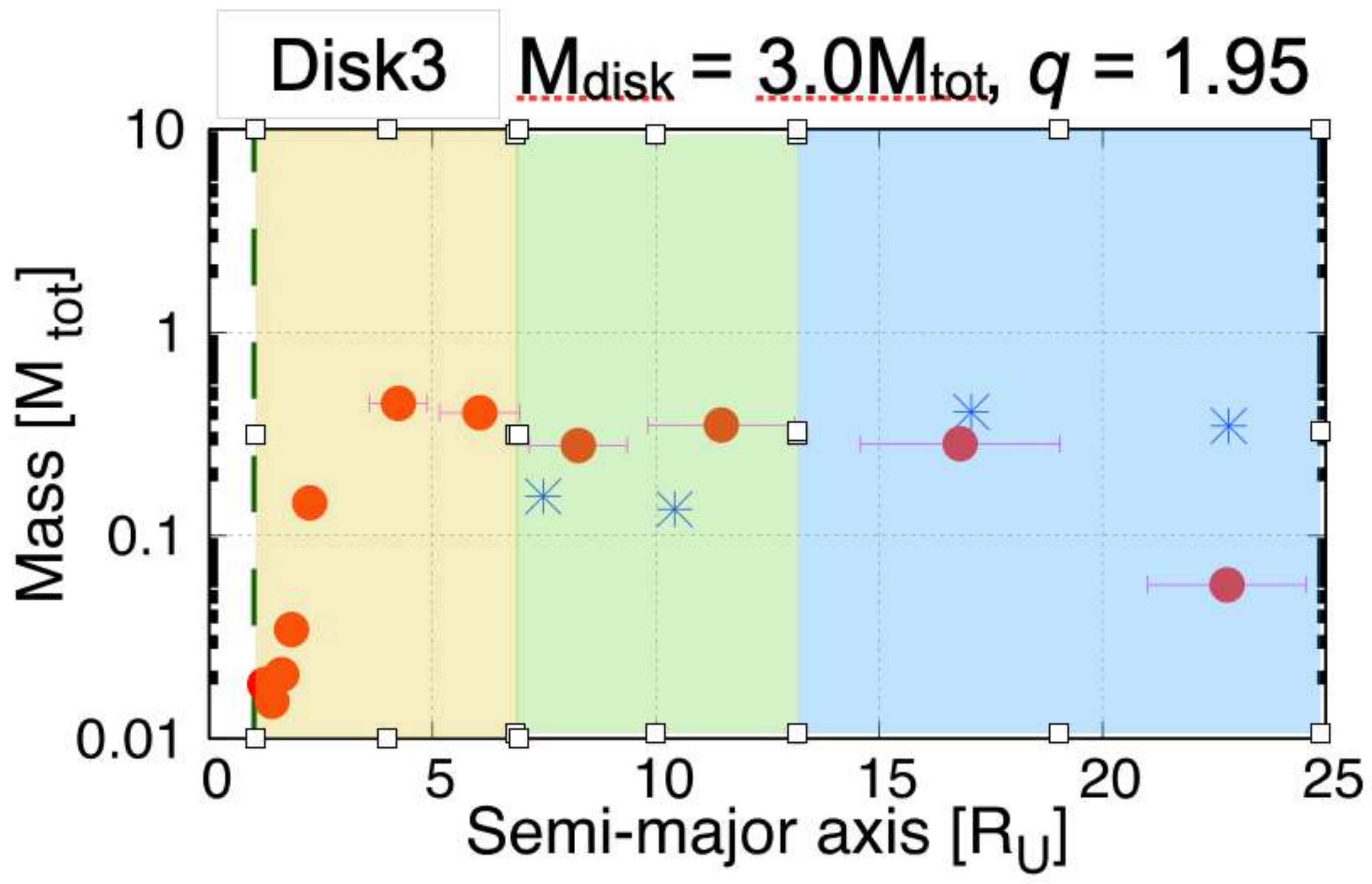
Results of Numerical simulations ●

The current satellites ✳

similar tendency in other disk models





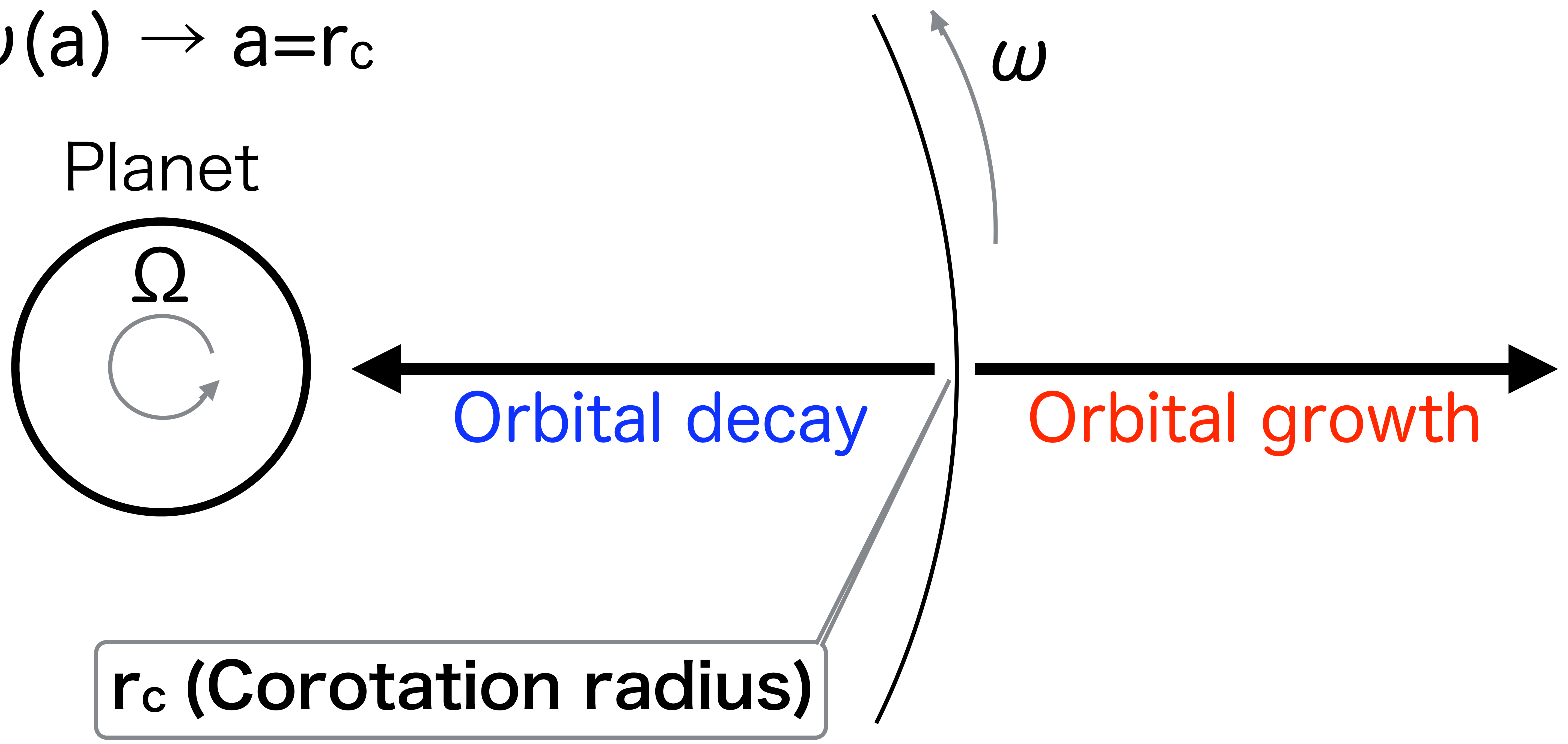


$R_U < a < 7R_U$	Extra massive satellites
$7R_U < a < 13R_U$	Larger satellites
$13R_U < a$	Smaller satellites

Orbital evolution by planet's tides

Corotation radius : the orbital radius where a satellite has an angular velocity equal to a spin angular velocity of a planet

$\Omega=\omega(a) \rightarrow a=r_c$



(Charnoz et al., 2011)

- Inside : $\Omega < \omega$, recieving negative torque ⇒ Orbital decay
- Outside : $\Omega > \omega$, recieving positive torque ⇒ Orbital growth

The corotation radius of Uranus $r_c \sim 3.3 R_U$

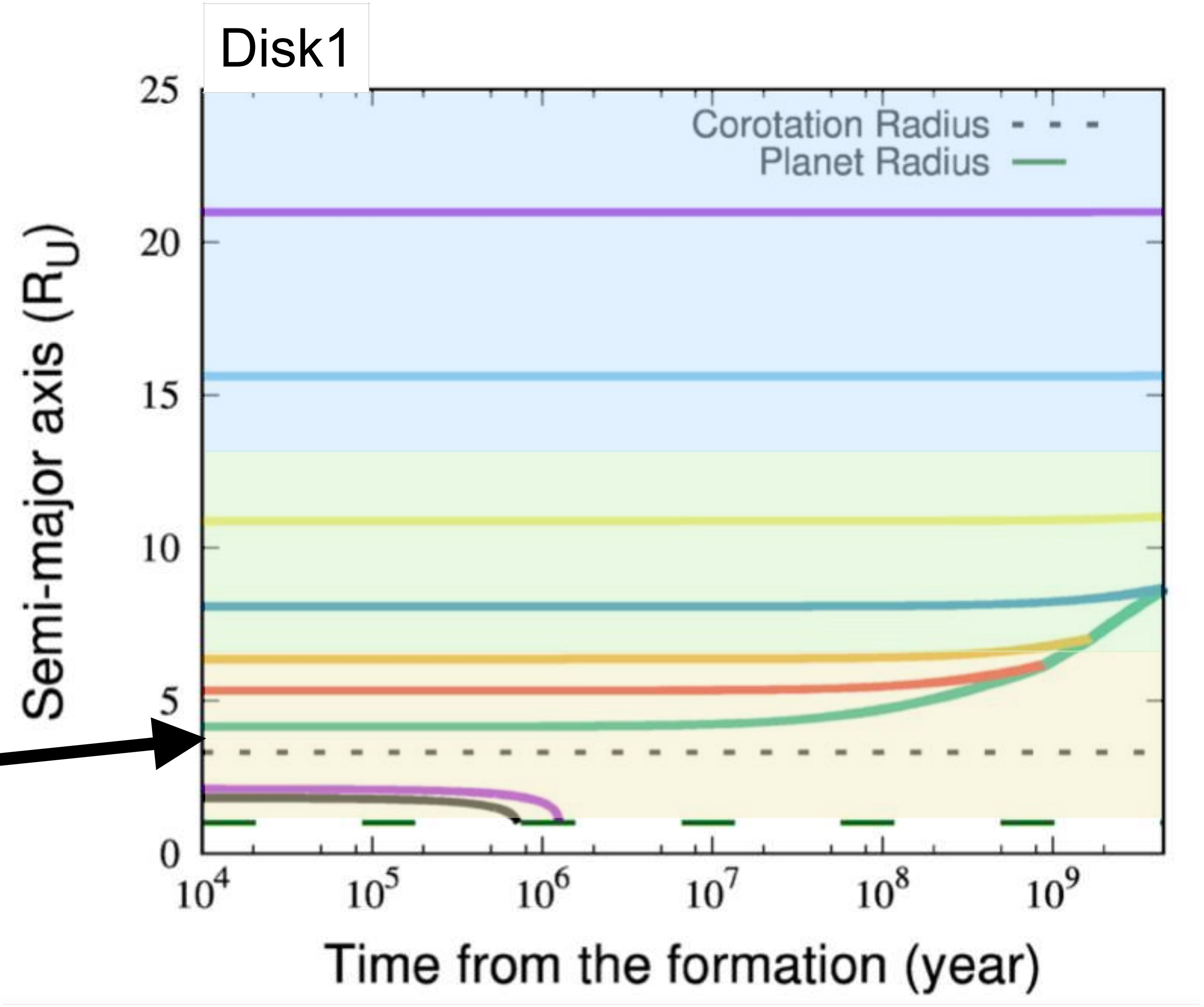
Orbital evolution of satellites by the planet's tides

A satellite's semi-major axis evolves according to

$$\frac{\mathrm{d}a_\mathrm{s}}{\mathrm{d}t} = \mathrm{sgn}(a_\mathrm{s} - r_\mathrm{c})\frac{3k_{2\mathrm{p}}M_\mathrm{s}G^{1/2}R_\mathrm{p}^5}{Q_\mathrm{p}M_\mathrm{p}^{1/2}{a_\mathrm{s}}^{11/2}}$$ (Charnoz et al. 2011)

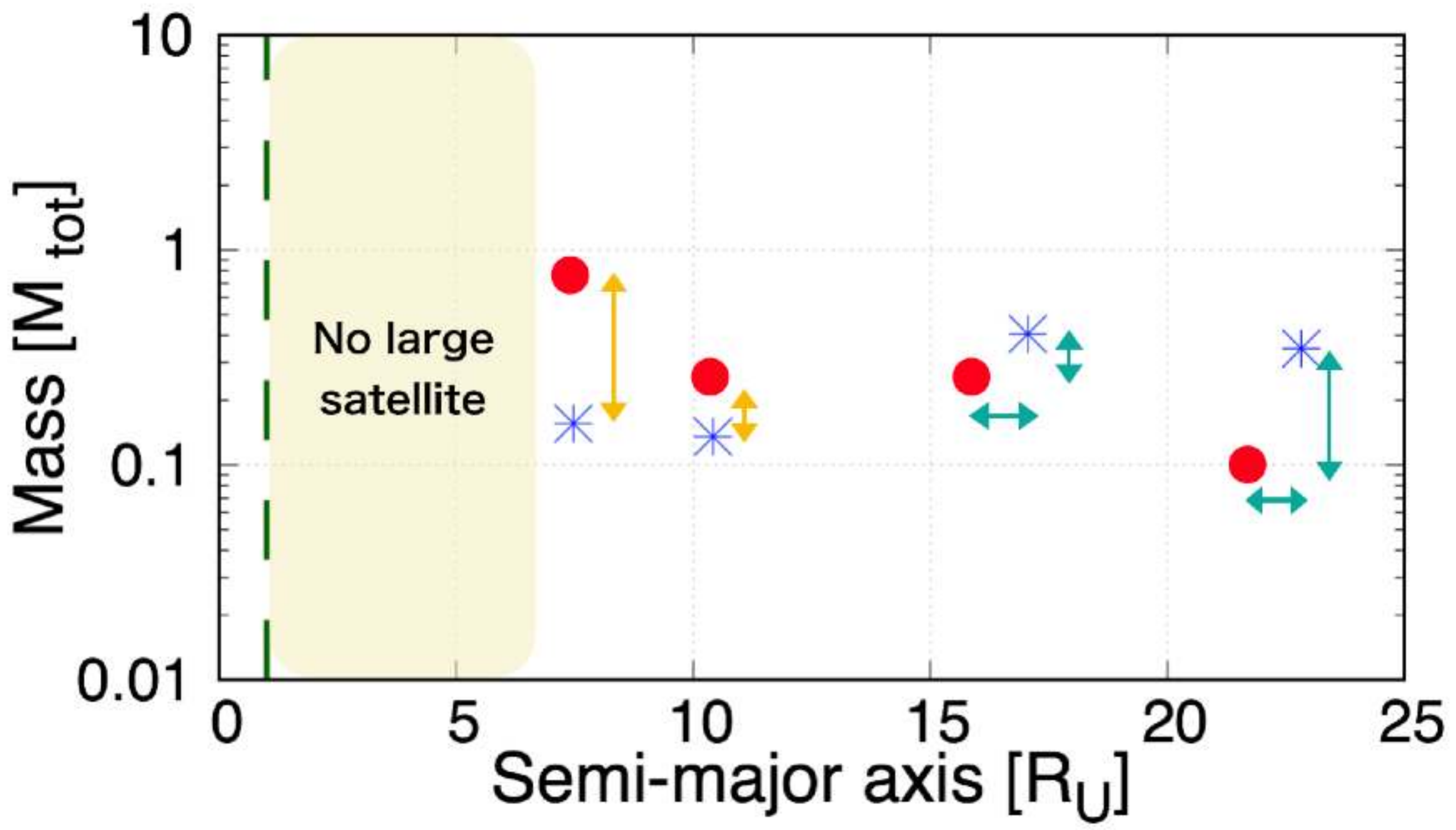
$k_{2p} = 0.104$ (Gavrilov and Zharkov 1977)

$Q_p = 11{,}000$ (Tittemore and Wisdom 1989)



The corotation radius of Uranus $r_c \sim 3.3R_U$

- Inner satellites fall into Uranus or move outward
- Satellites in the middle merge each other
- Outer satellites (>10RU) remain in the almost same orbits



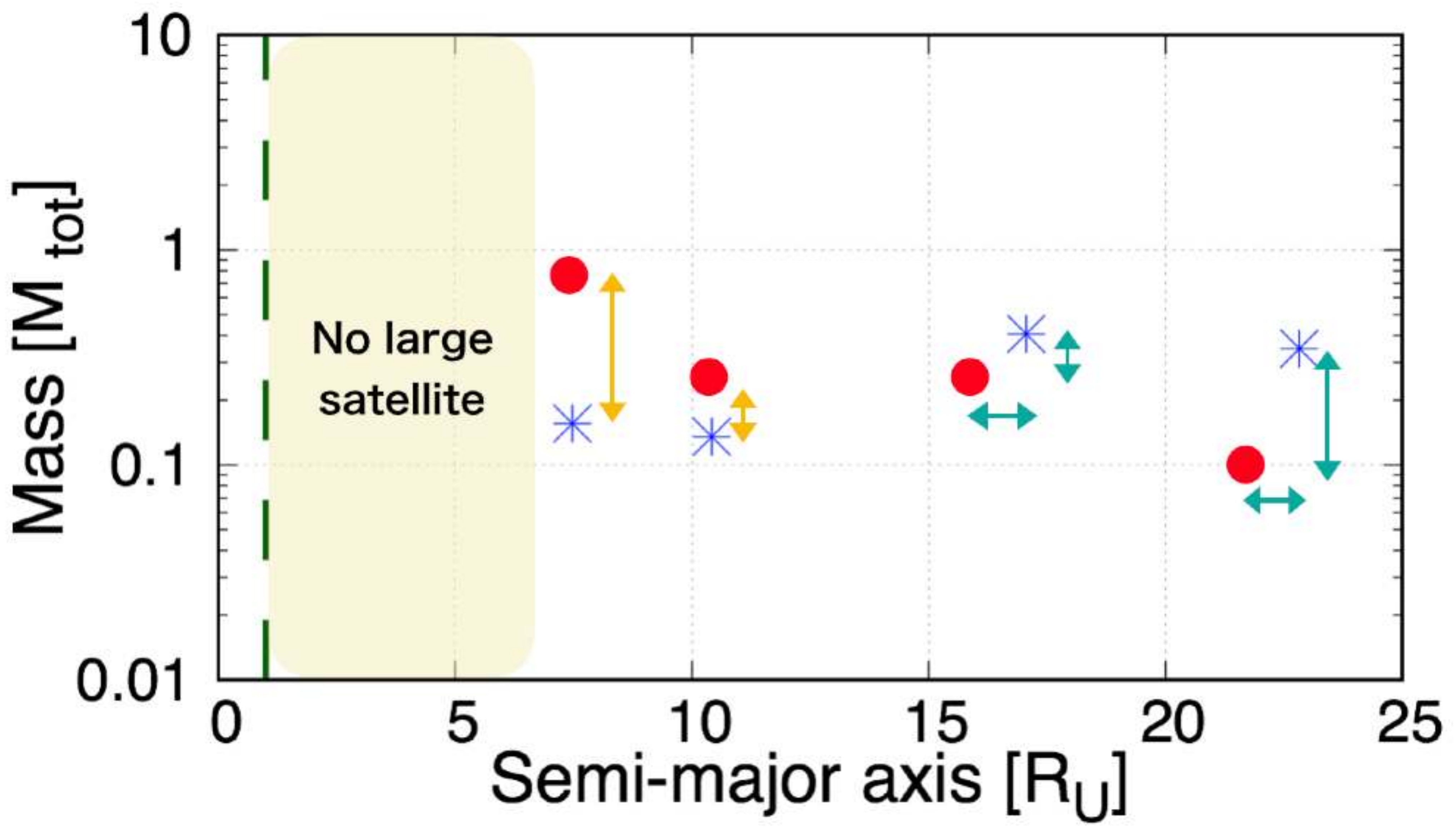
Even if orbital evolution is considered, the satellite distribution could not been reproduced from a disk with negative gradient

Ishizawa, Sasaki & Hosono (2019)

We performed *N*-body simulations and analytical calculations to investigate the possibility of the in-situ satellite formation from a debris disk around Uranus produced by GI.

When the disk surface density has a negative power law gradient,

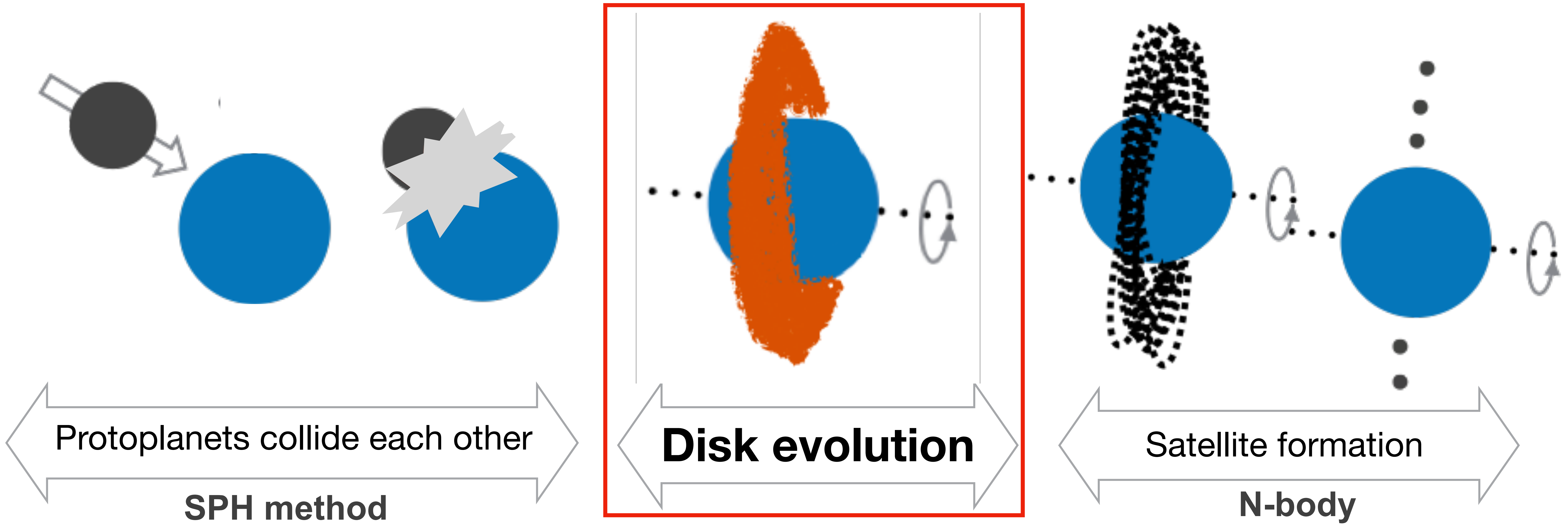
- satellites with too large masses would form around the orbit of Ariel an Umbriel
- the outermost satellites would not reach to the mass and the orbital radius of Oberon



It would be difficult to reproduce the distribution of the current satellite system from a debris disk with a negative gradient

Ida et al. (2020)

GI scenario for the uranian satellite formation



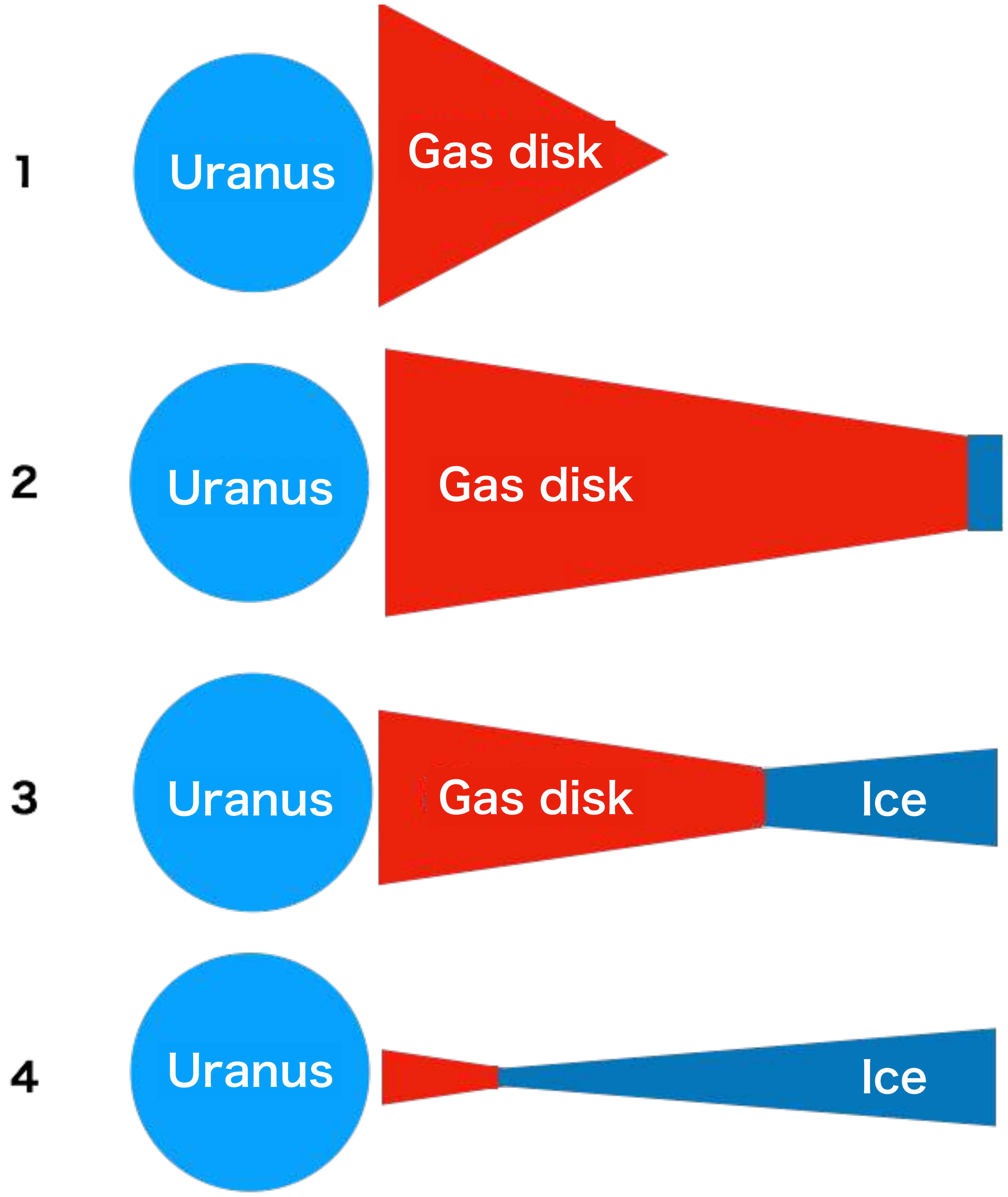
In Chapter 1,

- The current satellites could not form from a debris disk with a negative gradient
- The evolution of an impact-generated disk is not considered

✦ A disk generated by a giant impact is mostly vaporized
✦ It may undergo viscous diffusion until the re-condensation of materials

Here we investigated the evolution of the impact-generated disk of a mixture of water vapor and H/He gas until the disk cools down enough for ice condensation and performed an N-body simulation for the condensed ice disk.

Evolution model by Ida et al. 2020



An impact generates a gas disk of H/He and vaporized H_2O and rock

Viscous diffusion and radiative cooling of the gas disk

Ice condensates when T_{disk} falls bellow the freezing point of H_2O

A disk of ice has more mass on the outer side

Gas disk evolution model (Lynden-Bell & Pringle 1974; Hartmann+ 1998)

- **Viscosity *ν*** (Shakura & Sunyaev 1973)
  α is a constant parameter to represent the turbulence strength

$$\nu \sim \alpha c_s^2 \Omega^{-1}$$

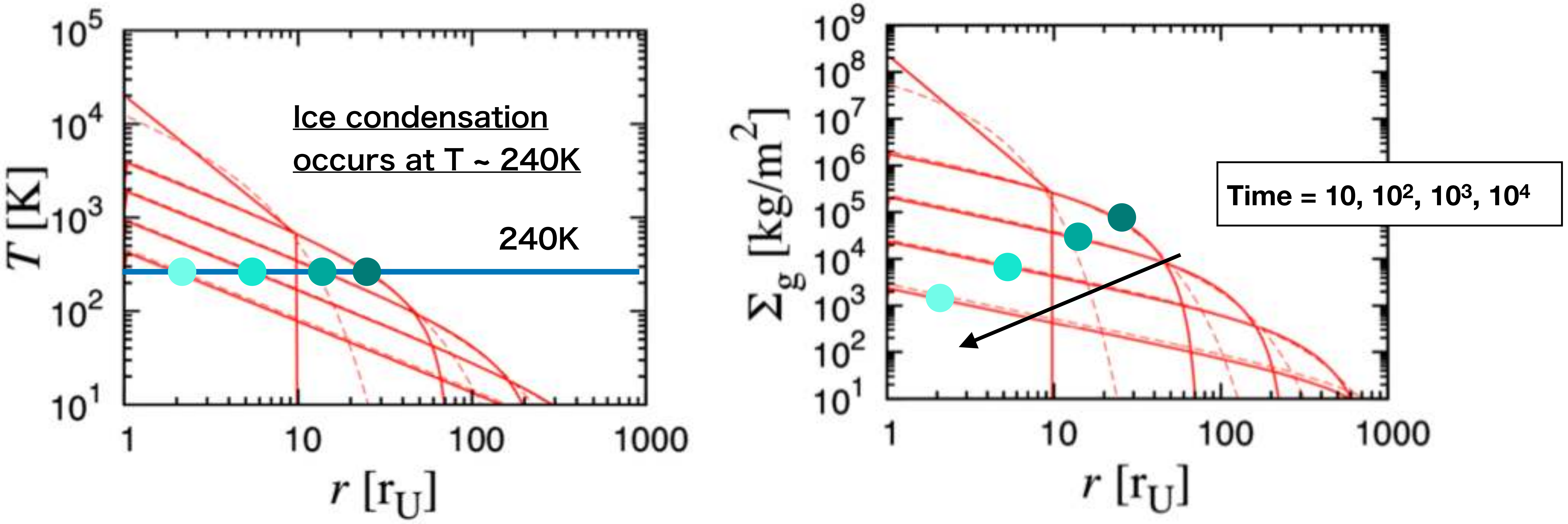
- **Local disk temperature *T***
  Equilibrium between viscous heating and radiative cooling

$$T \simeq \left(\frac{9\, G M_{\mathrm{U}} \Sigma_{\mathrm{g}} \nu}{8\, \sigma r^3} \right)^{1/4}$$

- **Surface density of gas (H/He/H_2O) Σ_g**
  Surface density evolves
  according to the viscous diffusion equation

$$\frac{\partial \Sigma_{\mathrm{g}}}{\partial t} - \frac{1}{r}\frac{\partial}{\partial r}\left[3 r^{1/2} \frac{\partial}{\partial r}\left(\Sigma_{\mathrm{g}} \nu r^{1/2} \right) \right] = 0$$

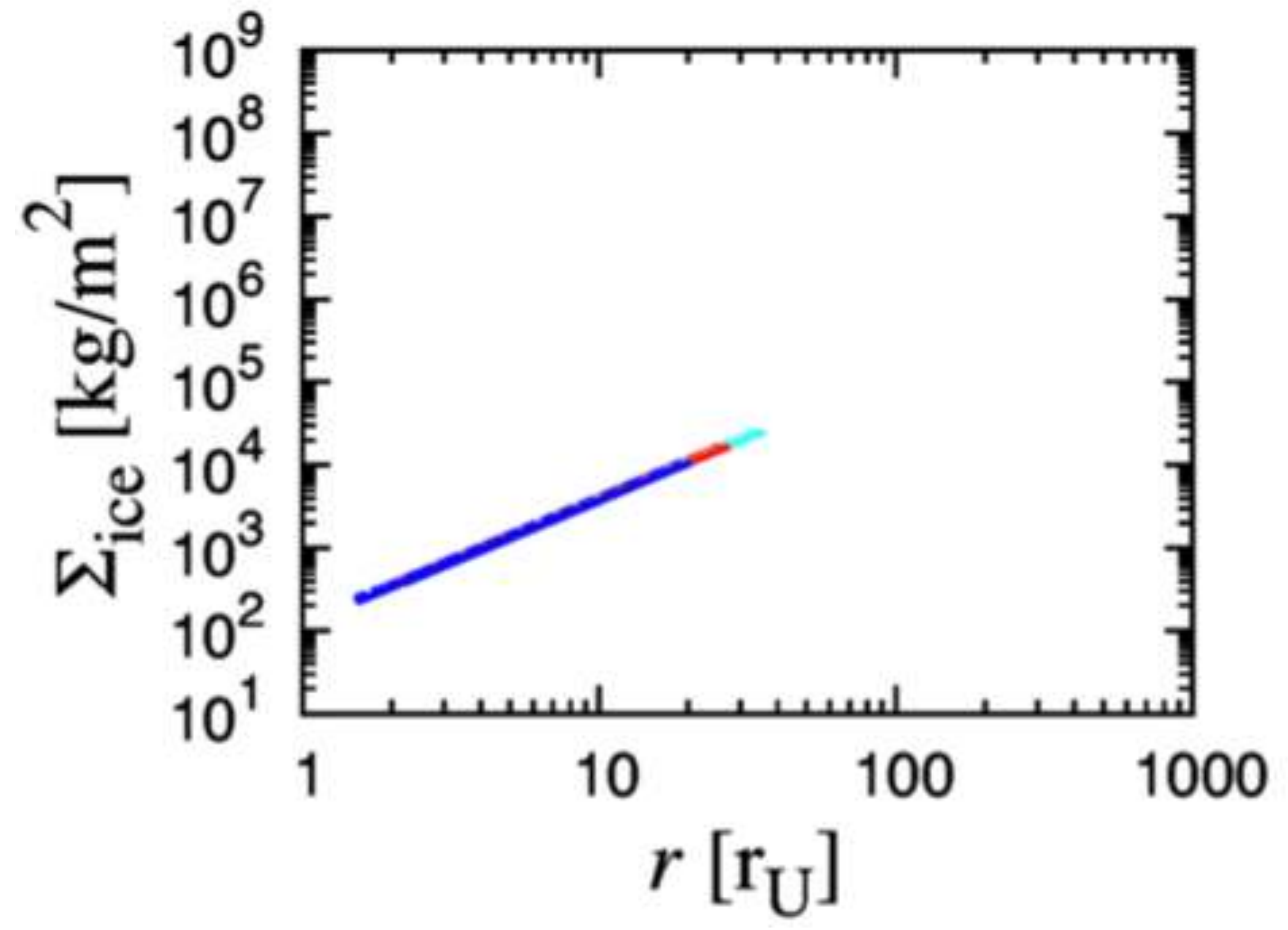
Ida et. al. (2020) performed the simulation
for the viscous diffusion of a vapor disk and also derived analytical formula



Disk model of condensed ice (Ida et al. 2020)

Ida et. al. (2020) found that
the surface density of the condensed ice
could have positive gradient

(almost independent of initial disk structure)



$$\Sigma_{ice} \simeq 1.2 \times 10^2 \left(\frac{\alpha}{10^{-3}}\right)^{-1} \left(\frac{T_{ice}}{240\mathrm{K}}\right)^3 \left(\frac{\Sigma_{H_2O}/\Sigma_{gas}}{0.3}\right) \left(\frac{r}{r_U}\right)^{3/2} \mathrm{kg/m^2}$$

$$r_{max} \simeq 20 \left[\beta \left(\frac{\langle r_{d,imp}\rangle}{2\,R_U}\right)^{-5/4} \left(\frac{M_{disk}}{10^{-2} M_U}\right)\right]^{1/4} R_U$$

We performed an N-body simulation for the condensed ice disk modeled by Ida et al. (2020), using a typical value suggested by the SPH simulations

N-body simulation for the disk with positive gradient

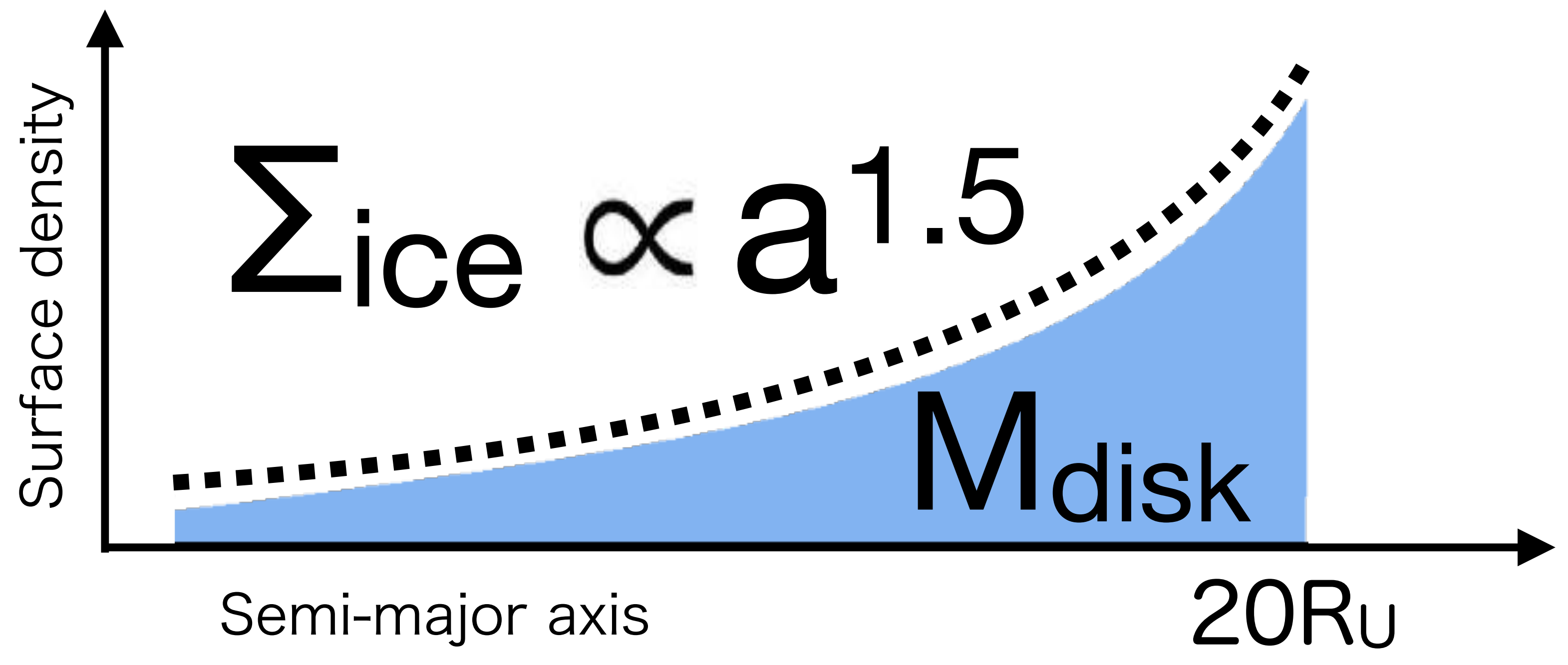
- **N-body simulation of accretion of condensed icy particles**
- **N-body simulations code is same as in Ishizawa+19**
- **The initial condition of the circumplanetary disk is derived from the results of Ida+20**

✦ Number of particles → 10,000

✦ Disk size a_{max} → $11R_U$, $20R_U$

✦ Disk mass M_{disk} → $0.92\ M_{tot}$

✦ Surface density $\Sigma(a) \propto a^{-q}$ → **-q = 1.5**

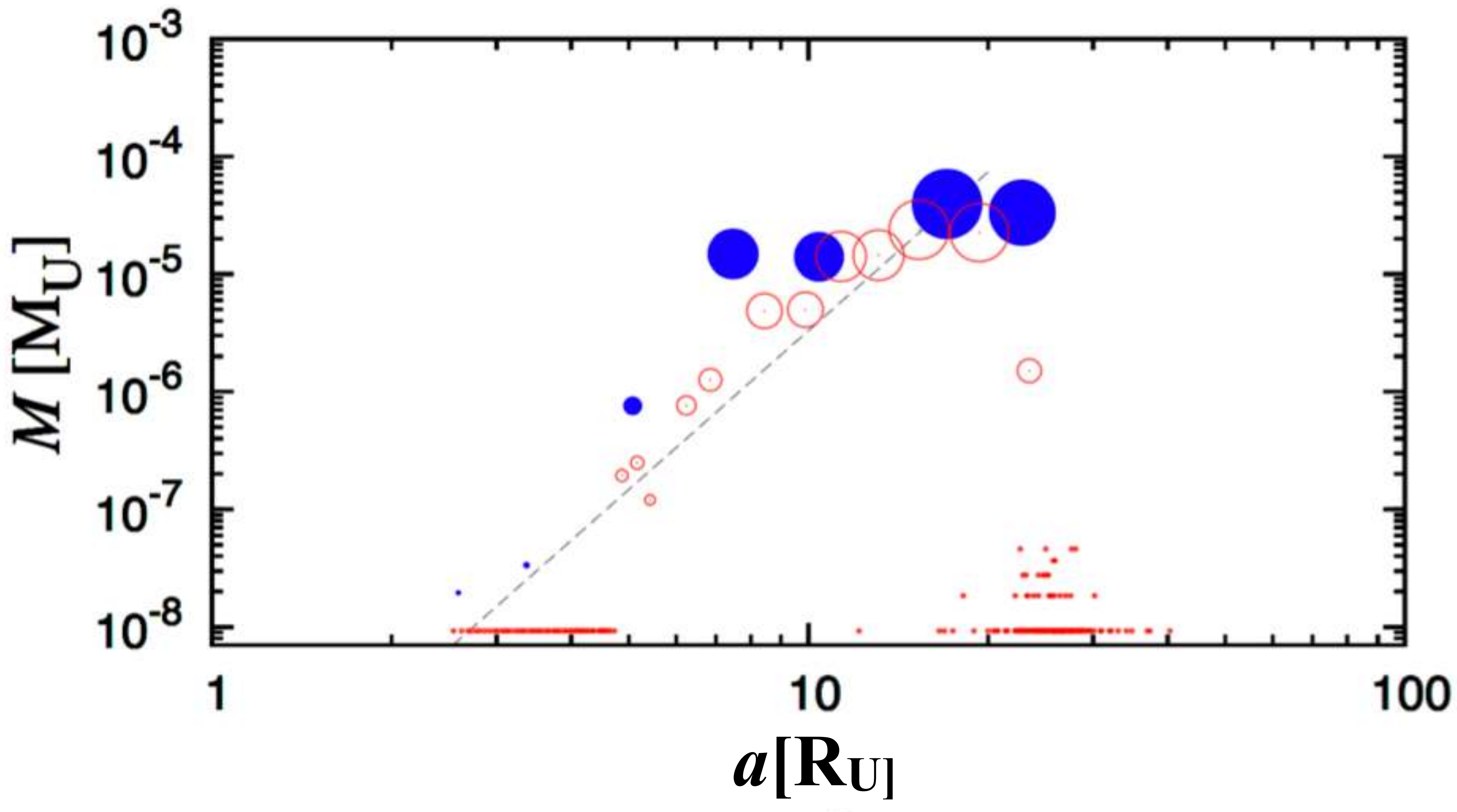


N-body simulations for the disk with positive gradient

- a_{max} = 20R_U

Results of Numerical simulations

The current satellites



- The results show grown-up particles at 280 years
- The outermost particle will accrete small particles to be Oberon's mass

The mass-orbit correlation of the satellites can be reproduced

Ida, Ueta, Sasaki & Ishizawa (2020)

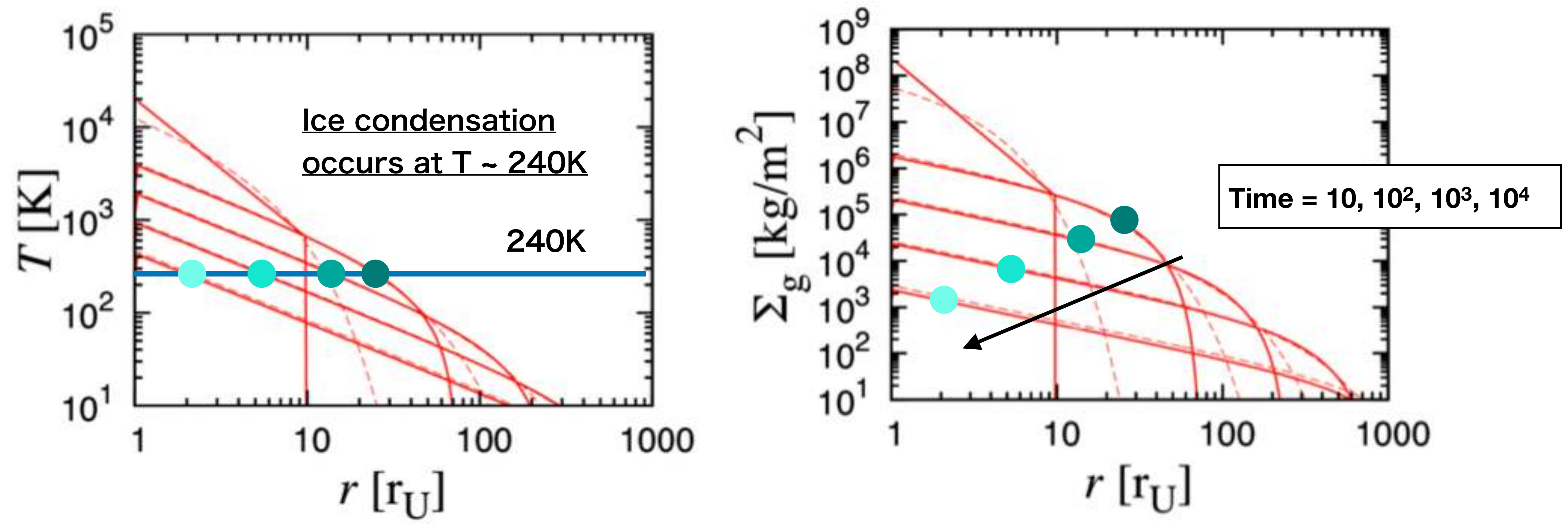
- Condensed ices are concentrated at ~ $20R_U$ and its total mass is ~10^{-4} M_U, almost independent of the initial disk structure
    - because the condensation temperature of water is low, the disk mass is significantly reduced until the ice re-condensation
    - **ice surface density has a positive gradient** as a result of equilibrium between viscous heating and radiative cooling

- **In the N-body simulation for the disk model by Ida et al. (2020) the mass-orbit distribution of the current satellites is well reproduced**

▶ Evolution of ice/silicate distribution taking into account of radial drift and growth of materials should be investigated in future

Kihara et al., submitted

Statistical N-body simulations based on Ida et al. (2020)



$$M_{\rm ice} \simeq 0.58 \times 10^{-4} \beta^{1/8} \left(\frac{\gamma}{0.3}\right) \left(\frac{\langle r_{\rm d,imp}\rangle}{2r_{\rm U}}\right)^{-35/32} \left(\frac{M_{\rm d,imp}}{10^{-2}M_{\rm U}}\right)^{7/8} M_{\rm U},$$

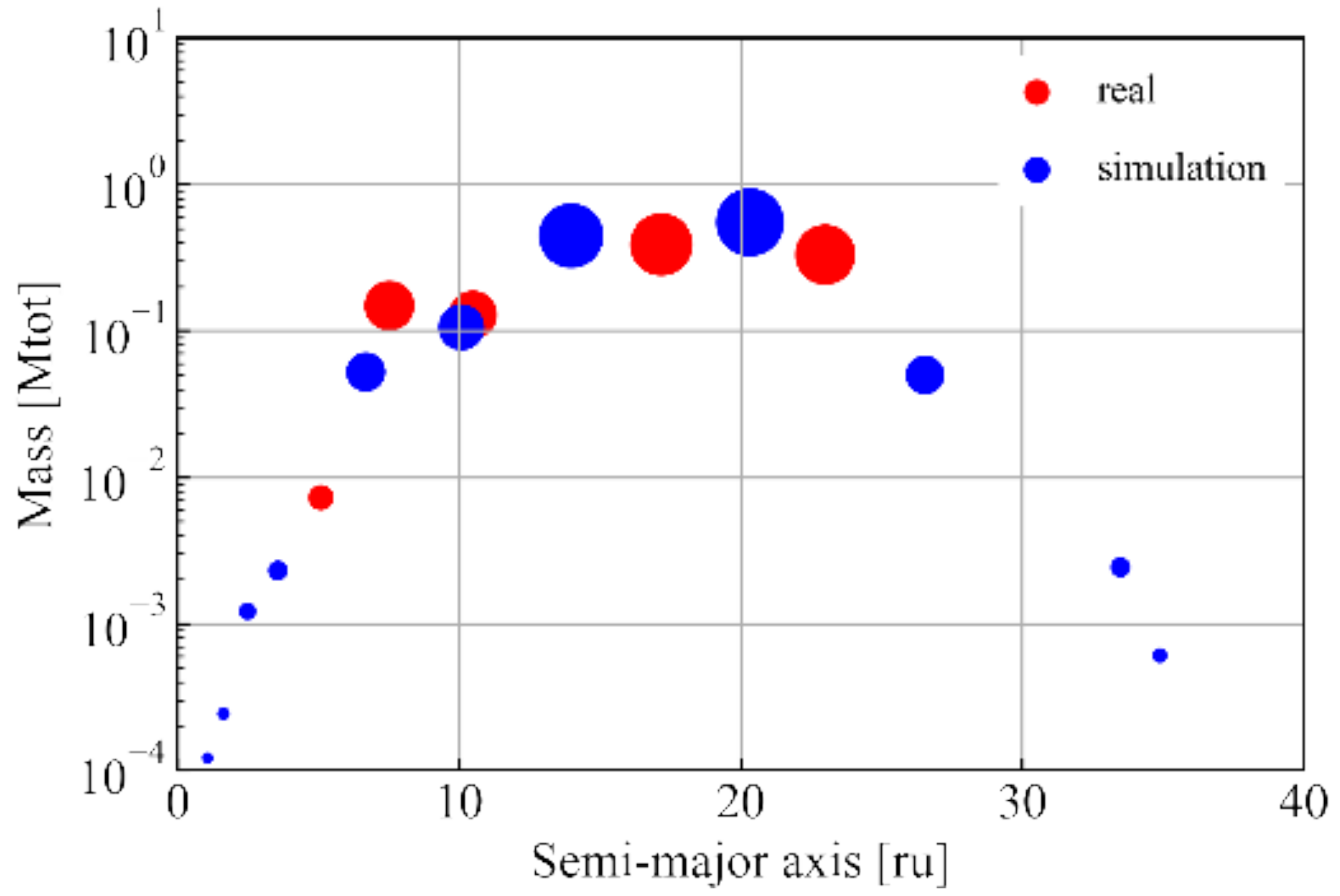
$$r_{\rm max} \simeq 20 \left[\beta \left(\frac{\langle r_{\rm d,imp}\rangle}{2r_{\rm U}}\right)^{-5/4} \left(\frac{M_{\rm d,imp}}{10^{-2}M_{\rm U}}\right)\right]^{1/4} r_{\rm U},$$

$$M_{\rm disk} = \frac{\rho_{\rm satellites}}{\rho_{\rm ice}} \frac{\rho_{\rm rock} - \rho_{\rm ice}}{\rho_{\rm rock} - \rho_{\rm satellites}} M_{\rm ice}$$

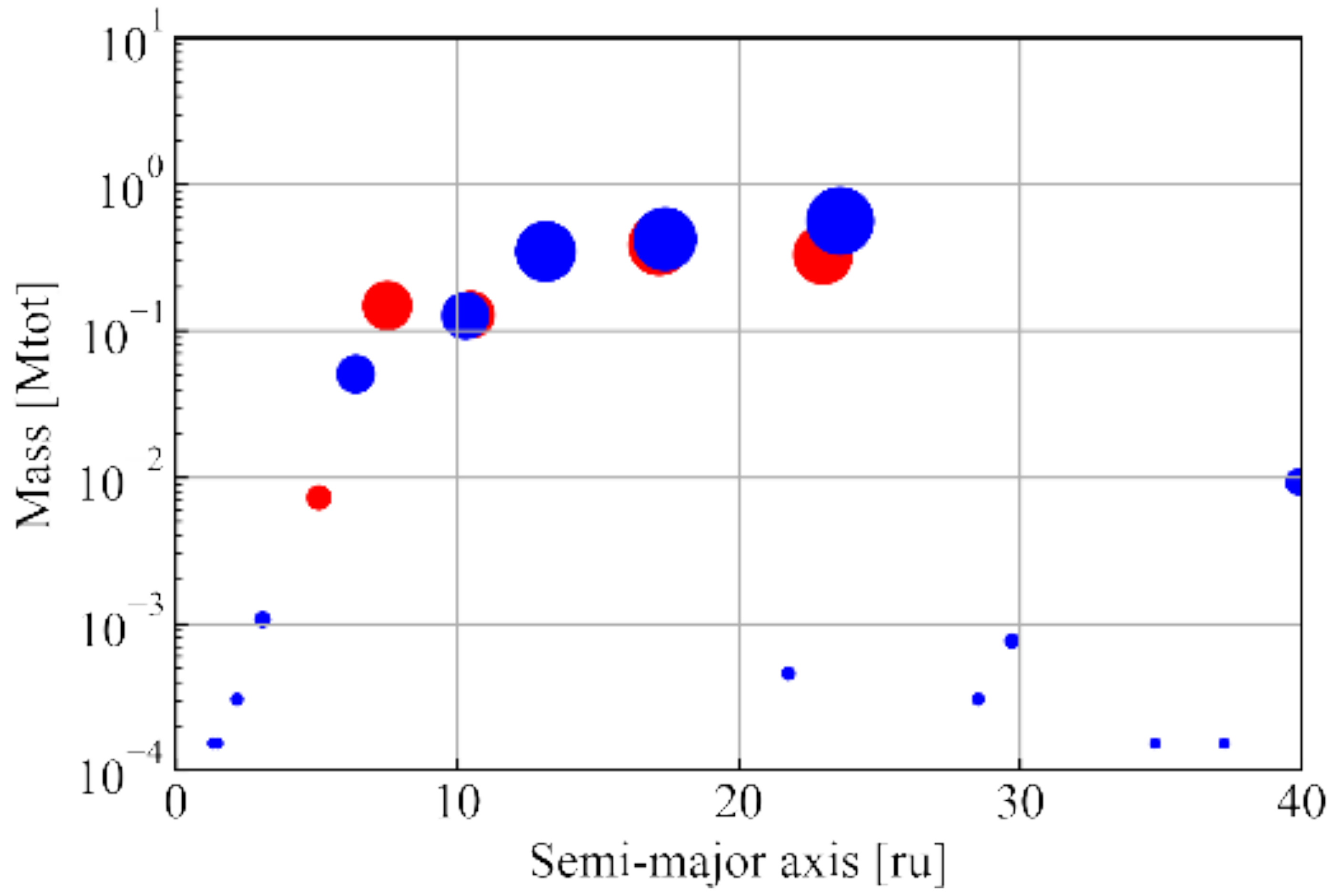
Table 1. Model sets of initial condition

Run	$M_{\rm disk}$ ($M_{\rm U}$)	$r_{\rm max}$ ($r_{\rm U}$)
1	1.02×10^{-4}	20
2	8.92×10^{-5}	19.3
3	7.95×10^{-5}	18.7
4	7.16×10^{-5}	18.1
5	6.51×10^{-5}	17.6
6	1.44×10^{-4}	22.1
7	1.27×10^{-4}	21.3
8	1.13×10^{-4}	20.6
9	1.02×10^{-4}	20.0
10	9.29×10^{-5}	19.5
11	1.86×10^{-4}	23.8
12	1.64×10^{-4}	22.9
13	1.46×10^{-4}	22.2
14	1.31×10^{-4}	21.5
15	1.19×10^{-4}	21.0
16	2.26×10^{-4}	25.1
17	1.99×10^{-4}	24.2
18	1.77×10^{-4}	23.5
19	1.60×10^{-4}	22.8
20	1.45×10^{-4}	22.2
21	2.65×10^{-4}	26.3
22	2.33×10^{-4}	25.4
23	2.08×10^{-4}	24.5
24	1.87×10^{-4}	23.8
25	1.70×10^{-4}	23.2

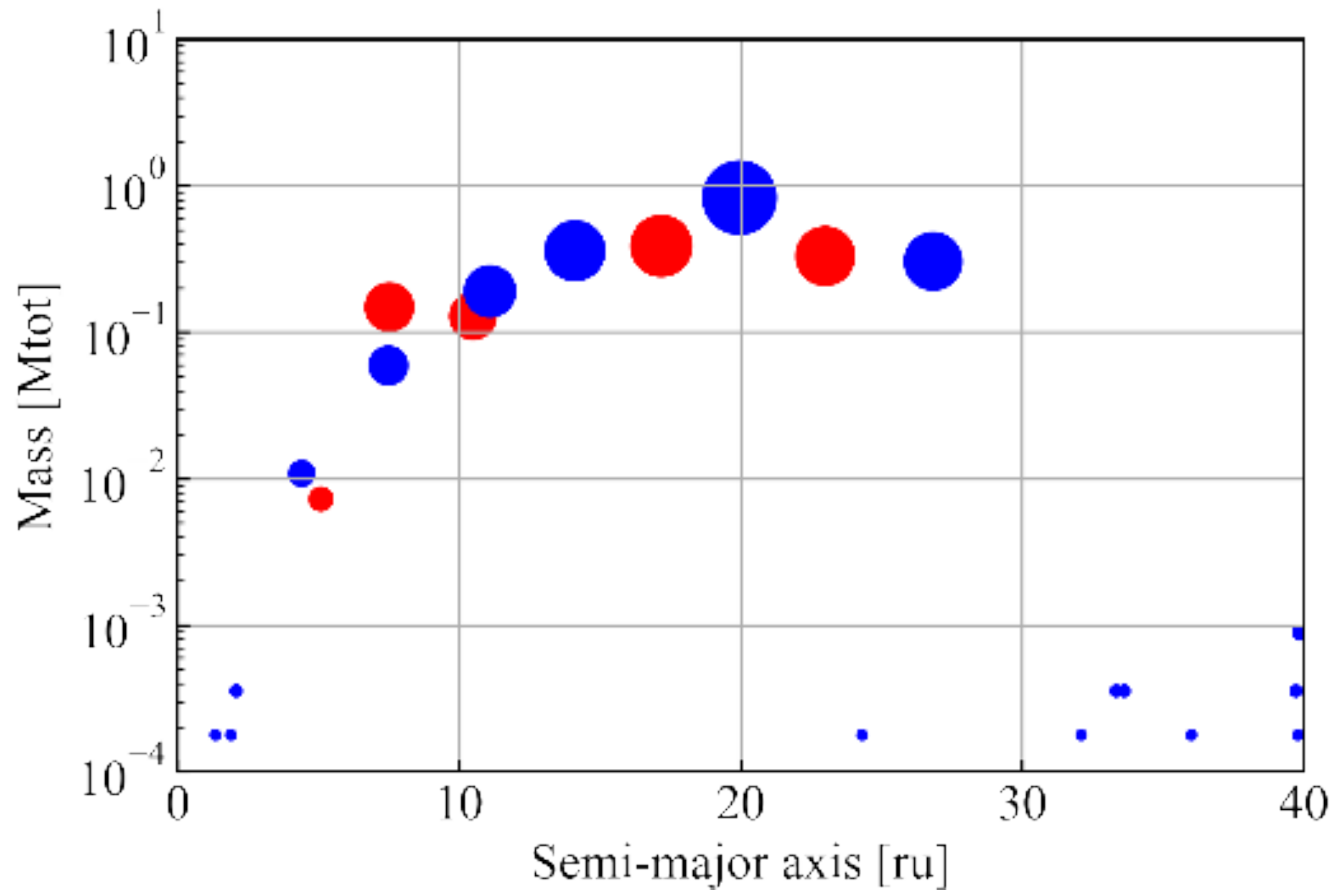
Typical results of N-body simulations



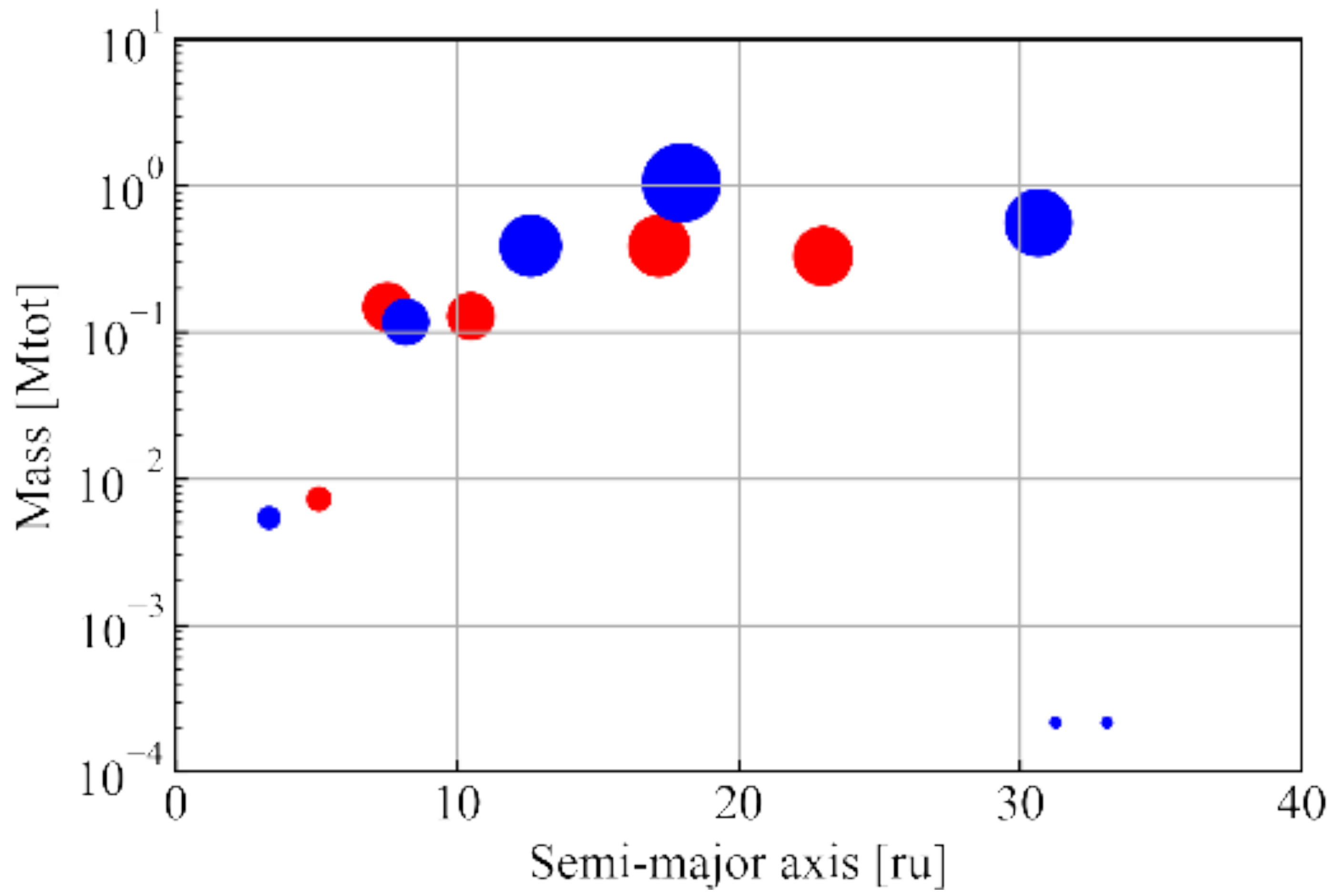
(a) The result of run7 at $4 \times 10^6 T_{\rm K}$



(b) The result of run19 at $8 \times 10^6 T_{\rm K}$

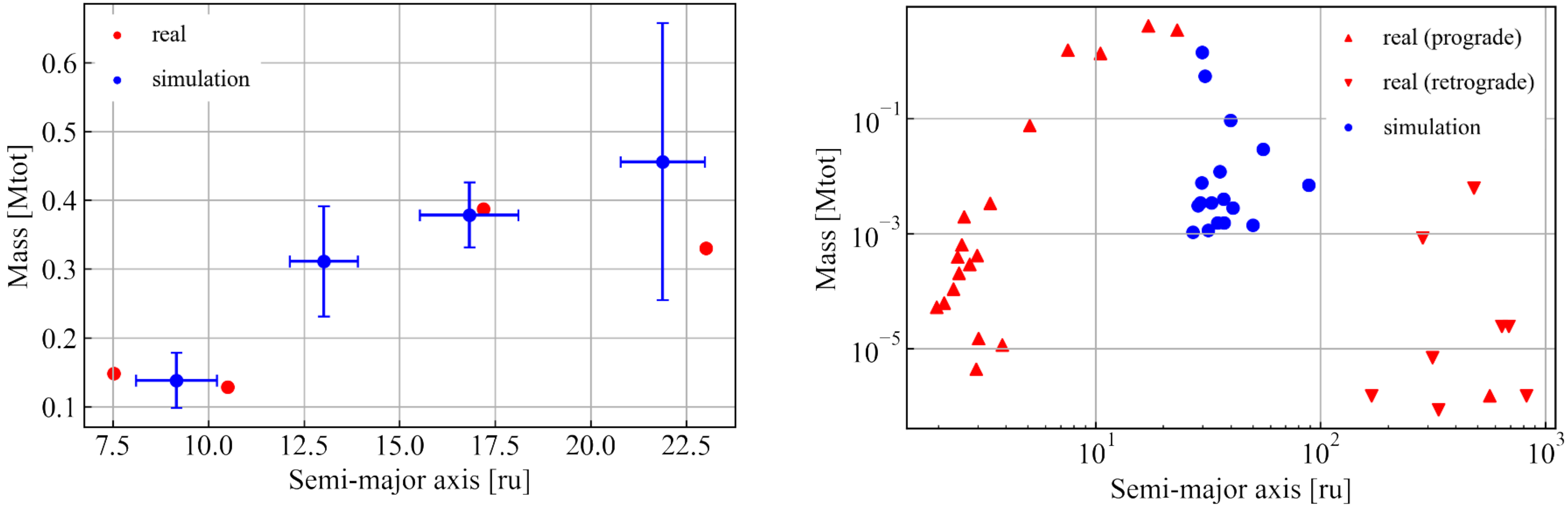


(c) The result of run11 at $5 \times 10^6 T_{\rm K}$



(d) The result of run16 at $10^7 T_{\rm K}$

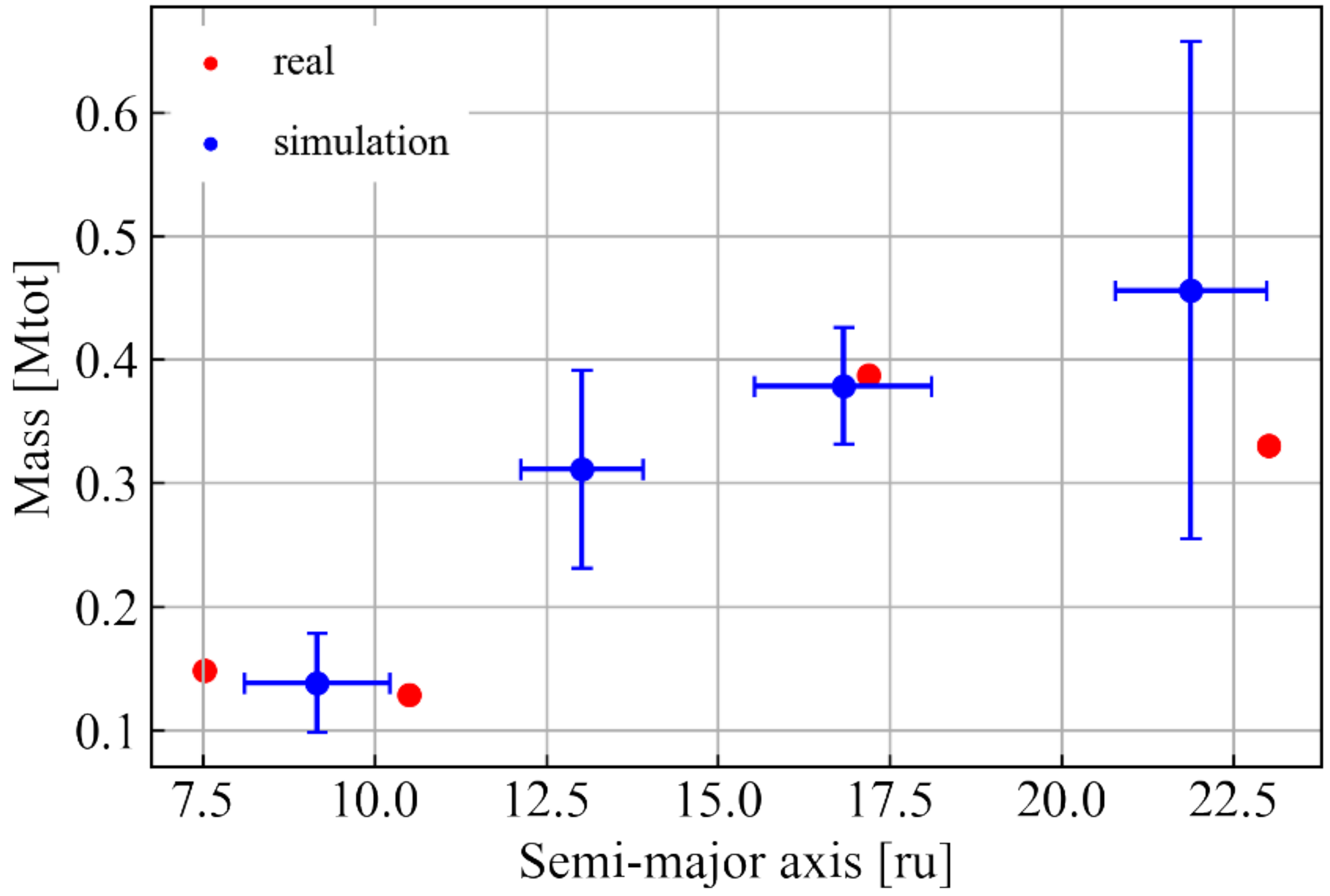
Statistical result of N-body simulations

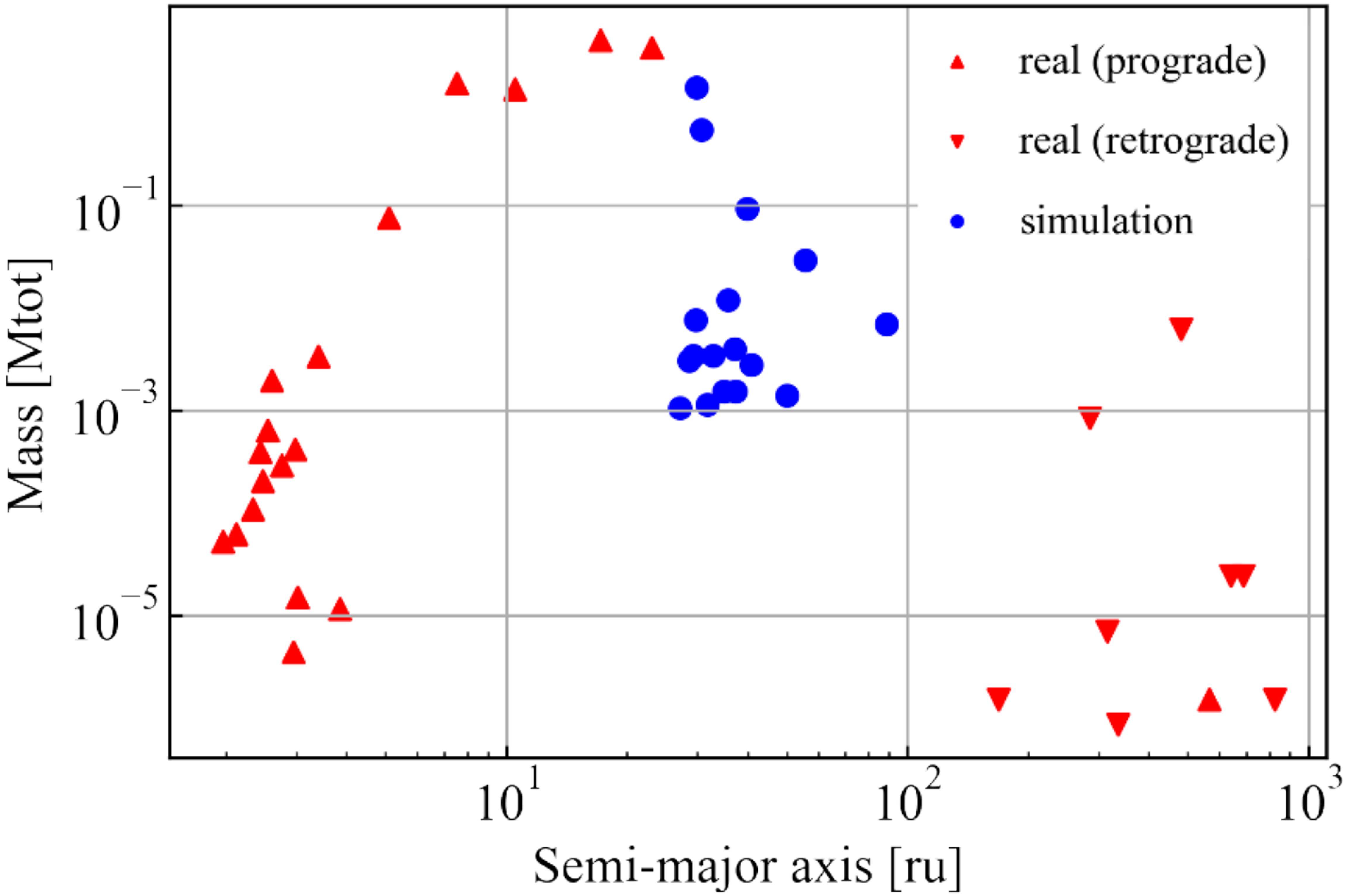


OK : Large 4 satellites are formed; smaller inside, larger outside

NG : Only 1 satellite is formed around 9 ru
NG : A large satellite is formed around 13 ru
NG : There are several satellites outside of Oberon's orbit

Statistical result of N-body simulations





Inner three satellites would be scattered with each other via orbital crossing on a timescale of ~ 10^6 years.

→ The satellites move into the current orbits?

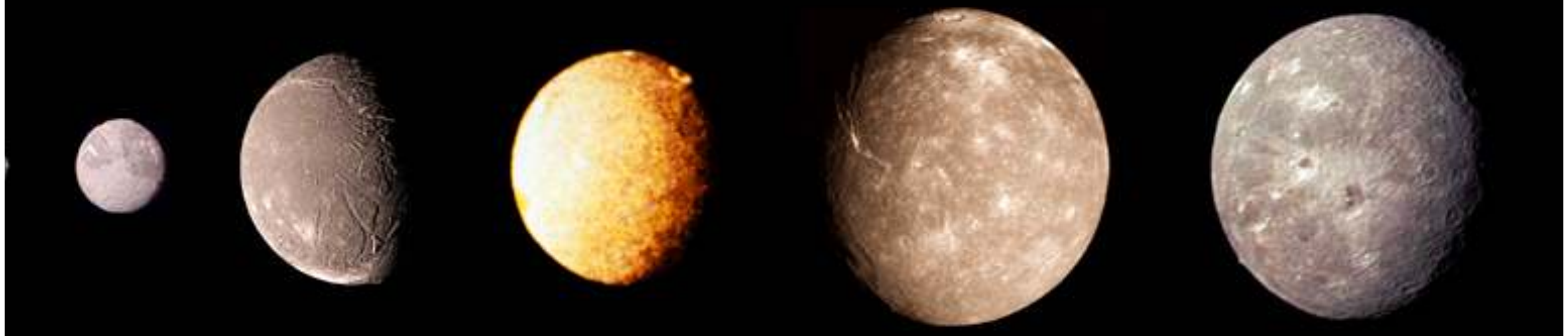
Protoplanets may pass through the vicinity of Uranus

→ It would scatter the outer moons to collide with Oberon?

Murashima & Sasaki, in prep.

Mean density(g/cm^3)

Miranda	Ariel	Umbriel	Titania	Oberon
1.21	1.67	1.40	1.72	1.63



Rock to ice mass ratio
Is roughly 0.5 ~ 1.0

(Hussmann+06)

© NASA/JPL Voyager 2

Rock content of the disk

- **The mass of the rock contained in the disk differs depending on the calculation**

The conditions of the calculations that have been performed are not uniform and the causes cannot be identified.

	EOS	Composition	Composition ratio	Impact Situation	Scheme	Resolution
Slattery 1992	SESAME ANEOS	Core (Iron) Core (Dunite) Mantle (Ice mix) Gas (Idel gas)	Iron / (Iron + Dunite) = 0.31 Ice / (Iron + Dunite) = 2.71 Gas : 2.0 Earth_Mass	Angular Momentum $(1 \sim 9) \times 10^{36}$	Standard SPH	8×10^3
Kegerreis 2018	Hubbard & MacFarlane 1980	Core (Iron + Dunite) Mantle (Ice mix) Gas (Ideal Gas)	Core : 2.02 Earth_Mass Mantle : 11.68 Earth_Mass Gas : 0.84 Earth_Mass	Angular Momentum $(1 \sim 9) \times 10^{36}$	Standard SPH	10^5, 10^6
Reinhardt 2020	Tillotson Ideal gas	Core (Granite) Mantle (Ice) Gas (Ideal gas)	Core : Mantle = 1 : 9 Gas : 2.0 Earth_Mass	Impact parameter $0.1 \sim 0.9$	ISPH + Density correction	10^5, 10^6 5×10^6

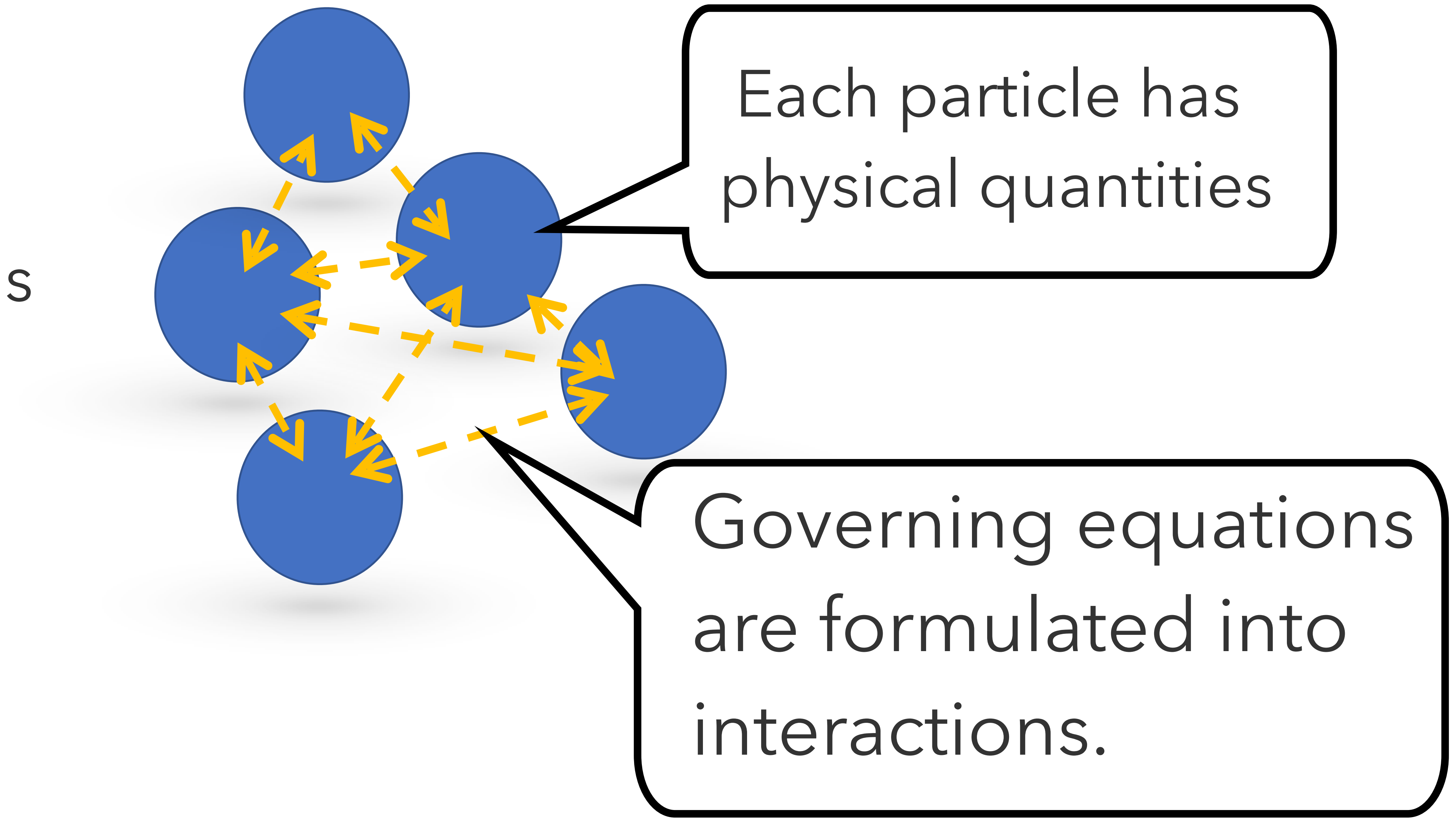
Simulate giant impacts in the Uranus system using three different EOS, investigate the effect of the different EOS on the characteristics of the disks formed.

Our Calculation

EOS	Composition	Composition ratio	Impact Situation	Scheme	Resolution
SESAME ANEOS	Core (Dunite) Mantle (Ice mix) Gas (Idel gas)	Ice / Dunite = 2.71 Gas : 2.0 Earth_Mass	Angular Momentum $(1 \sim 9) \times 10^{36}$	Standard SPH & DISPH	10^5
Hubbard & MacFarlane 1980	Core (Iron + Dunite) Mantle (Ice mix) Gas (Ideal Gas)	Iron / (Iron + Dunite) = 0.31 Ice / (Iron + Dunite) = 2.71 Gas : 2.0 Earth_Mass	Angular Momentum $(1 \sim 9) \times 10^{36}$	Standard SPH & DISPH	10^5
Tillotson Ideal gas	Core (Granite) Mantle (Ice) Gas (Ideal gas)	Iron / (Iron + Dunite) = 0.31 Ice / (Iron + Dunite) = 2.71 Gas : 2.0 Earth_Mass	Angular Momentum $(1 \sim 9) \times 10^{36}$	Standard SPH & DISPH	10^5

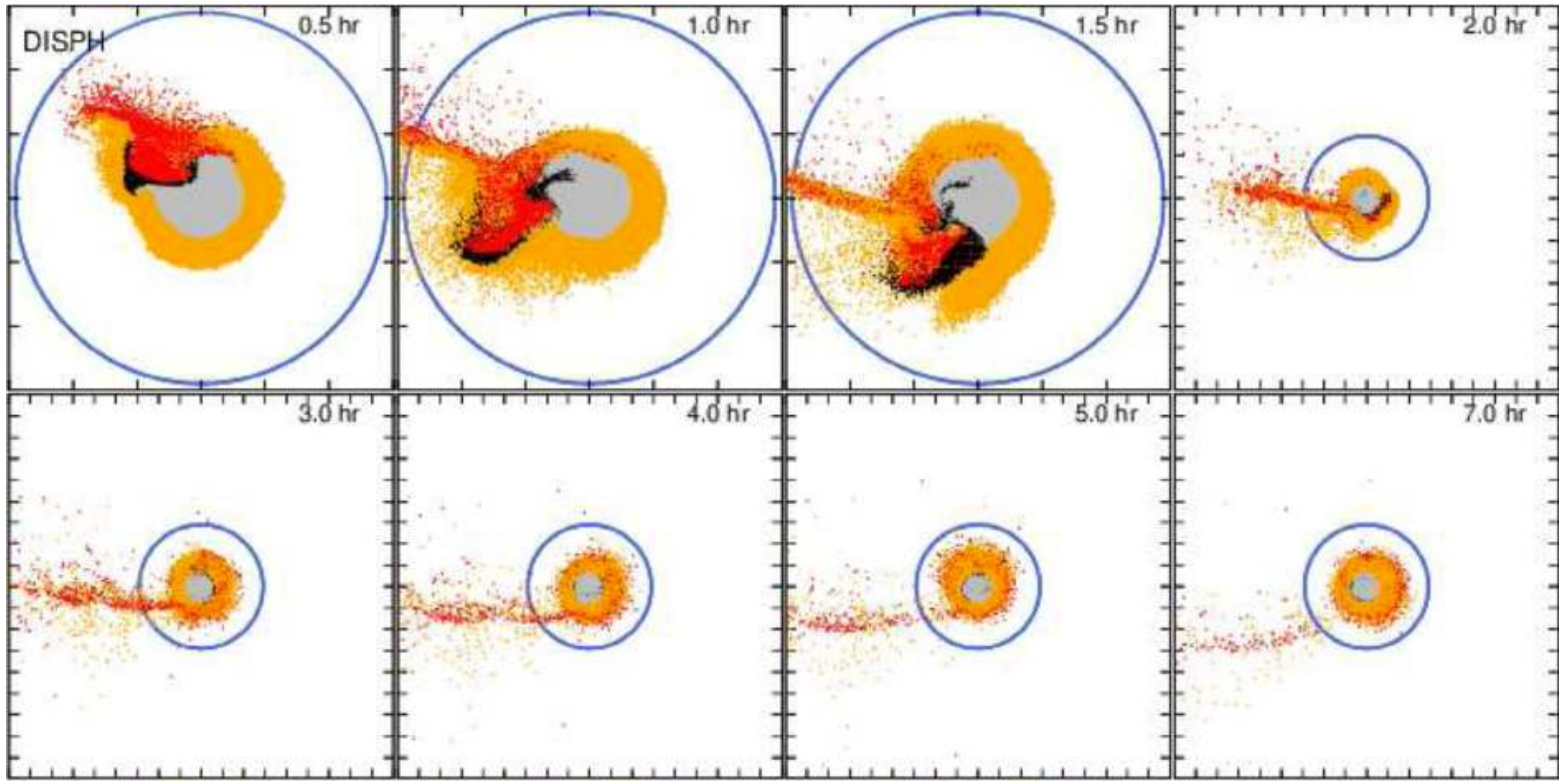
Smoothed Particle Hydrodynamics (SPH)

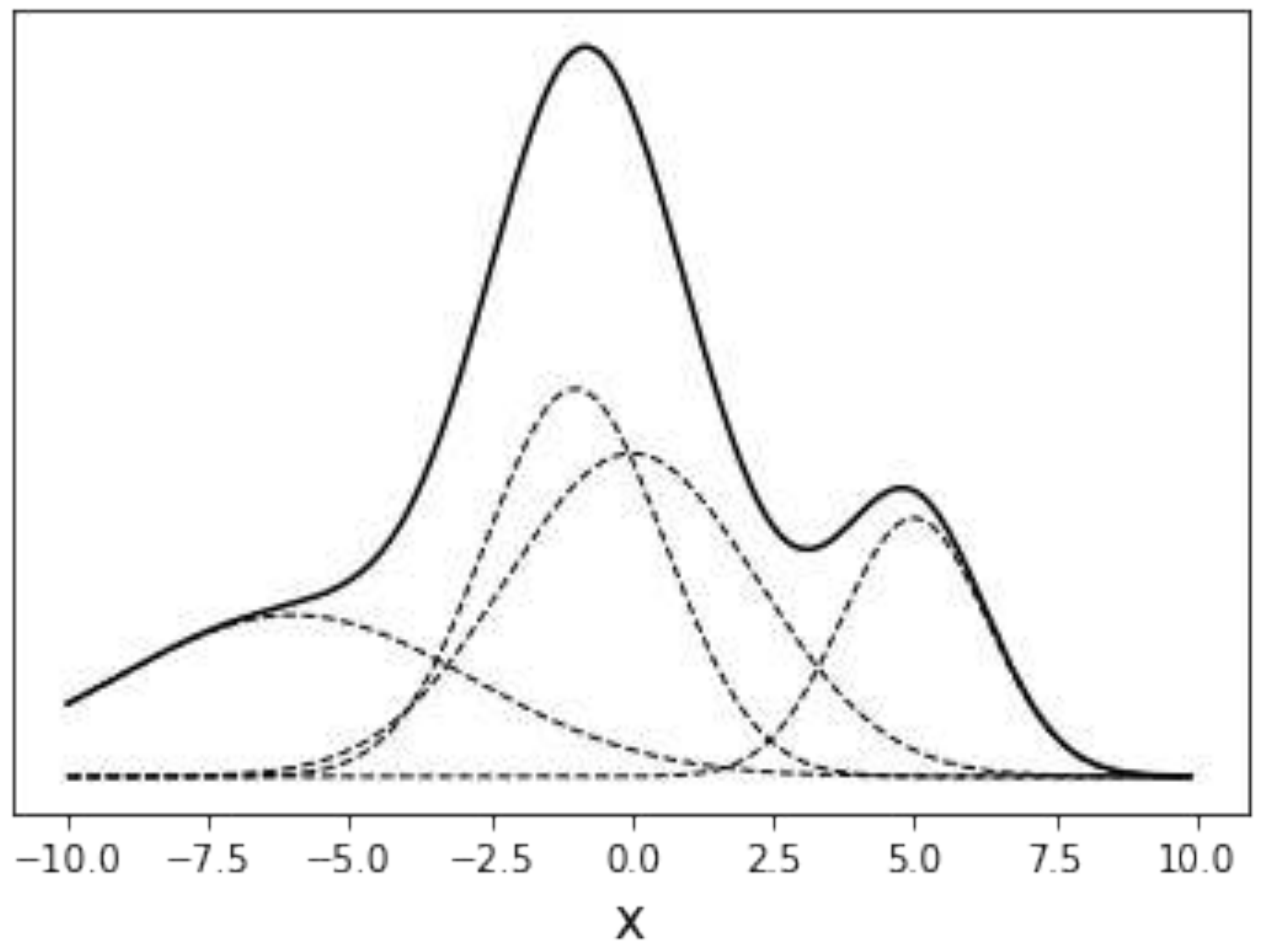
SPH was developed to simulate phenomena in astrophysics by Lucy (1977) and Gingold & Monaghan(1977).
In SPH, each particle has physical quantities and governing equations are formulated into interactions between surrounding paticles.



<u>**Examples**</u>…

Giant Impact (Hosono et al., 2016)

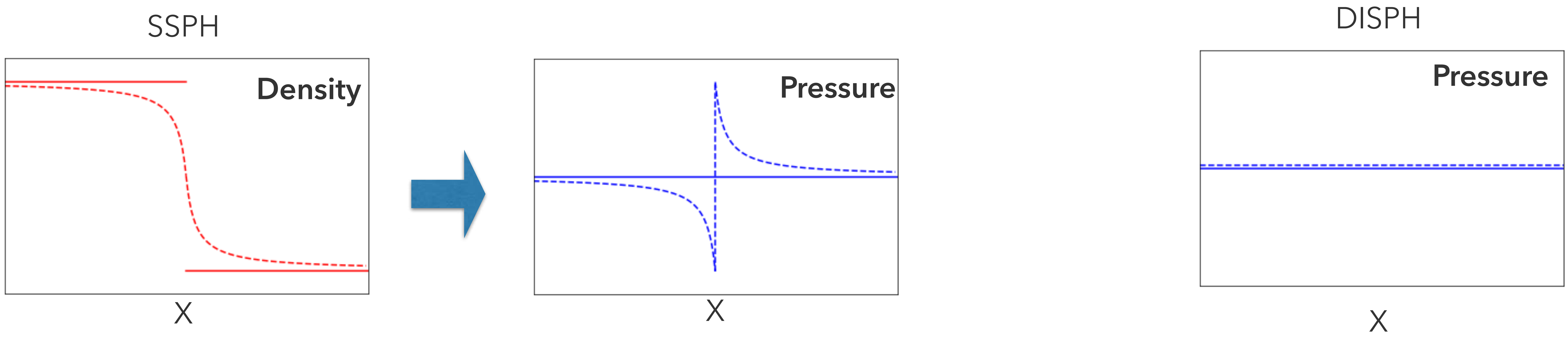




DISPH

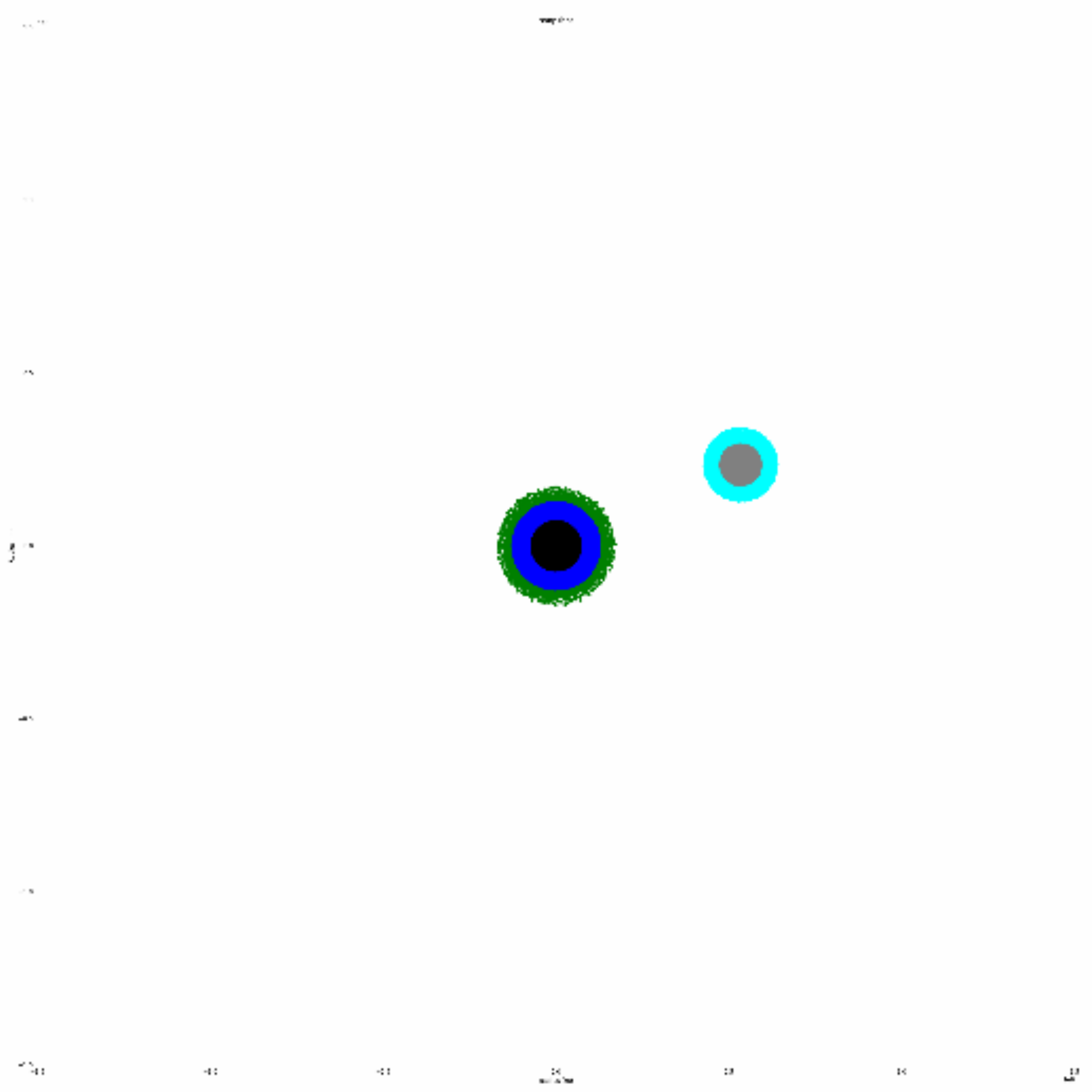
Density Independent SPH (DISPH) (Saitoh & Makino, 2013; Hosono et al., 2013)

"Instead of the mass density, we adopt the internal energy density (pressure) and its arbitrary function, which are smoothed quantities at the contact discontinuity, as the volume element used for the kernel integration.
We call this new formulation density-independent SPH (DISPH)."



Calculation condition

- Standard SPH and Density Independent SPH
- Total Mass : 14.5 M_{earth}
- Impactor Mass : 3 M_{earth}
- End time : 72 hours or 144 hours
- Angular Momentum : $1 \sim 9 \times 10^{36}$ kg m^2 s^{-1}
- Number of particles : $\sim 10^5$
- Target : rock+ ice (1 : 2.71) + atmosphere (2 M_{earth})
- Impactor : rock + ice (1 : 2.71)
- EOS : Tillotson, Hubbard & MacFarlane (1980), SESAME + ANEOS



Black : Rock(Target)
Gray : Rock(Target)
Blue : Ice(Impactor)
Sky blue : Ice(Impactor)
Green : Gas

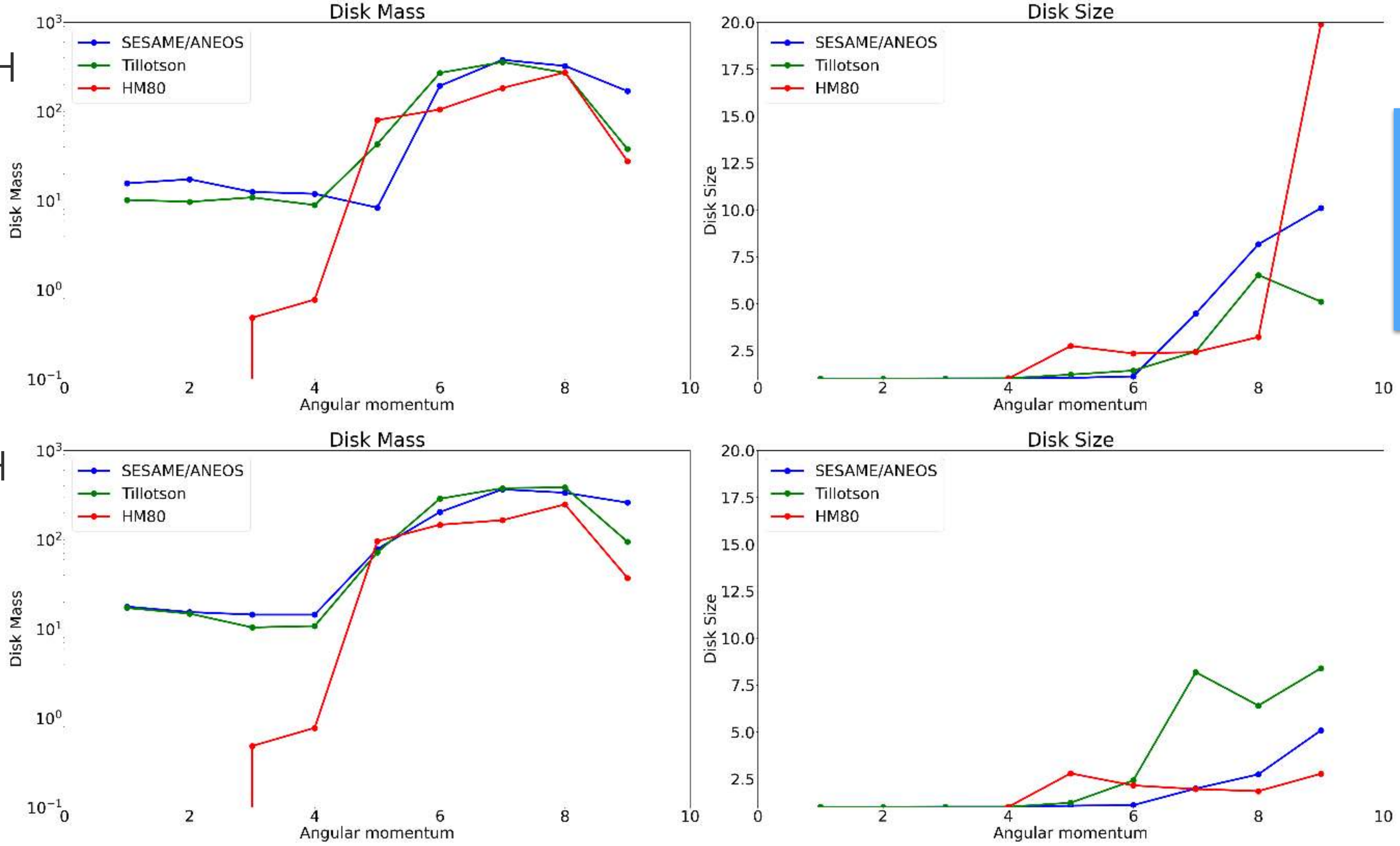
Rotation Period

- For calculations with more initial angular momentum than the current Uranus system, differences in the equation of state have little effect on the results

Mass and Size of disks

DISPH

SSPH



Disk Size

$$< r_{d,imp} > = \left(\frac{J_{d,imp}}{M_{d,imp} r_U^2 \Omega_U} \right)^2 r_U$$

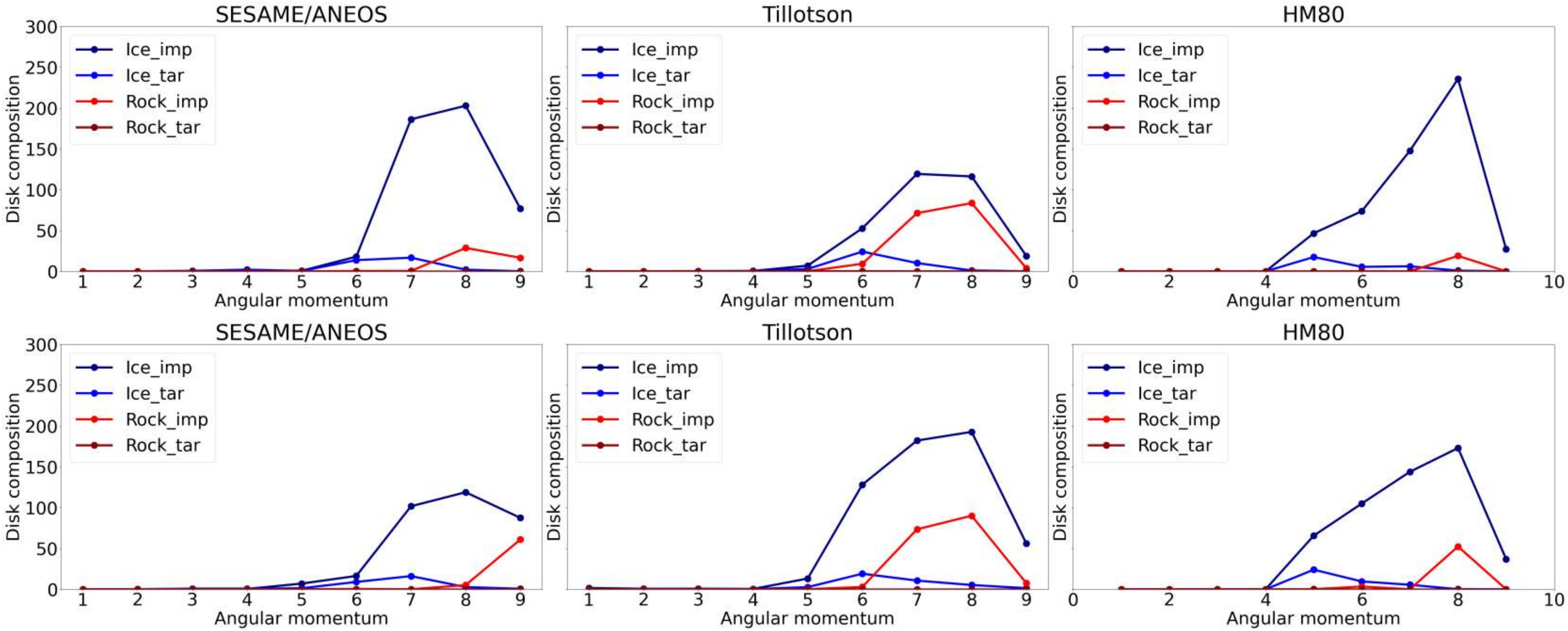
- The mass and size of the disk changes only a few times

Consistent with previous research

Mass of rock and ice in the disk

DISPH

SSPH



- Mass of rock in the disk　：　SESAME+ANEOS、HM80 < Tillotson

- The mass fraction of rock in the disk was similar to that of current satellites (30~50%) only when the initial angular momentum was high in the Tilloson EOS.

Murashima & Sasaki, in prep.

We simulated giant impacts of icy planet with three different equations of state (ANEOS/SESAME, Tillotson, and HM80) with Standard SPH and DISPH

- The mass, size, and Roation Period of the disk are hardly affected by the equation of state and scheme
- The mass of rock in the formed disk in high angular momentum : **SESAME+ANEOS,HM80 < Tillotson**
- There is no critical difference between schemes.
- Under the conditions of the present calculations, the mass ratio of rock in the disk is similar to that of the current satellite (30~50%), but only when the initial angular momentum is high in Tilloson EOS.

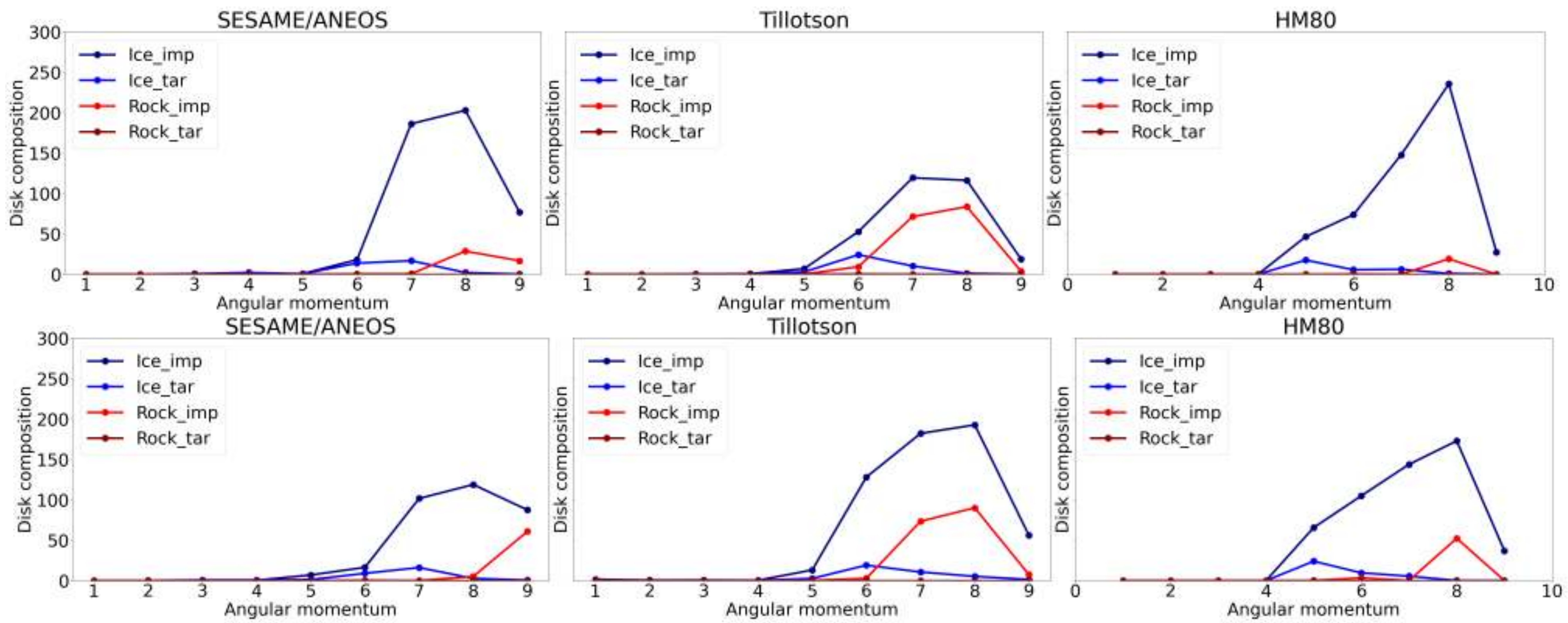
Explanation of the characteristics of Uranus' current moons requires impact of rock-rich impactors, contamination by KBOs, etc., or rock enrichment during the evolutionary stages of the disk.

Future work

- <u>Calculate disk evolution using disk properties obtained from SPH calculations</u>

DISPH

SSPH



1D semi-analytical calculation & N-body simulation

(Ida et al. 2020, Woo et al. 2022, etc…)

Satellite systems at each **equation of state**, **SPH scheme**, and **initial angular momentum**