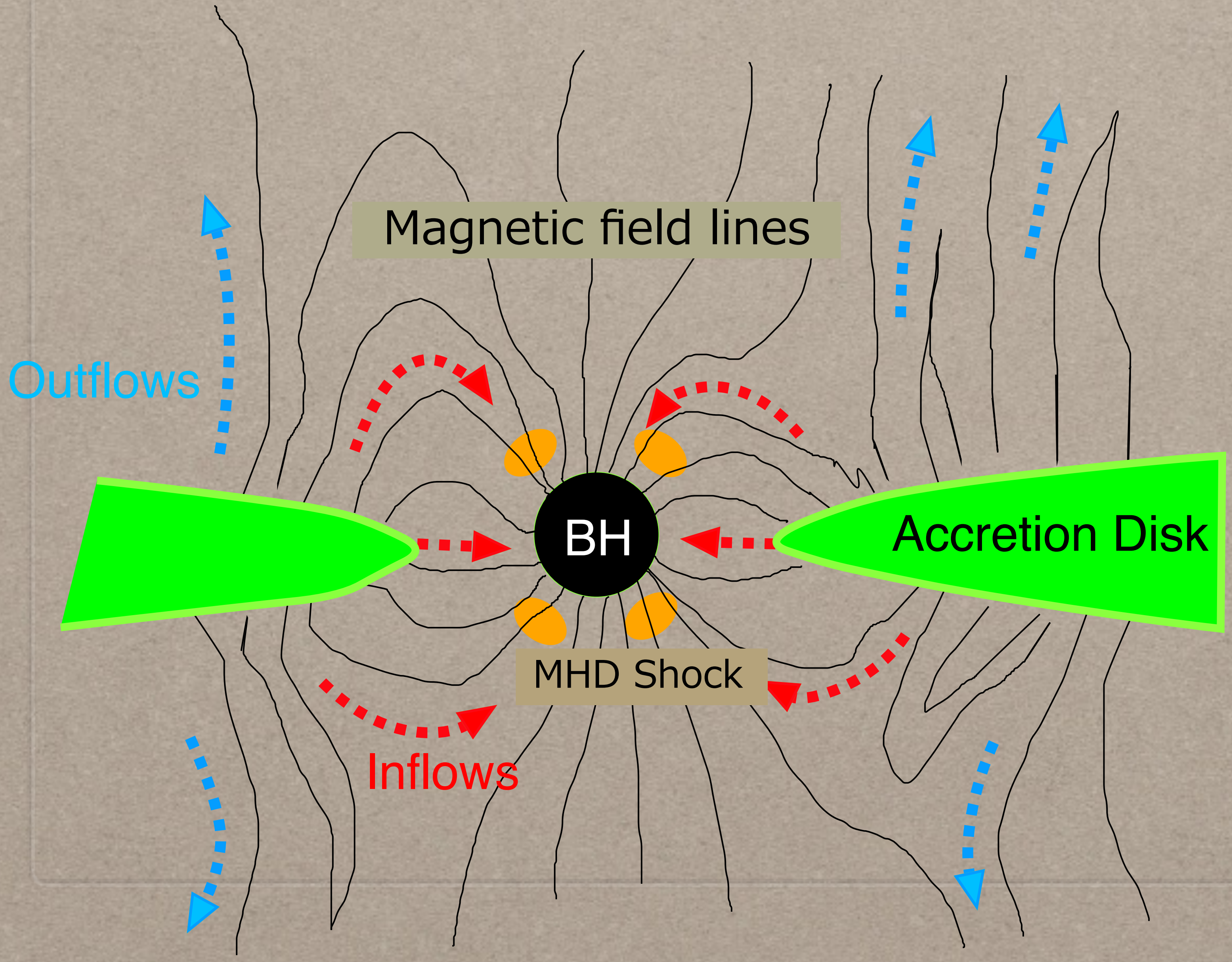
降着円盤大研究会@京都大学 2015/06/21-22

ブラックホール磁気圏の 磁場構造と宇宙ジェット形成



高 橋 真 聡

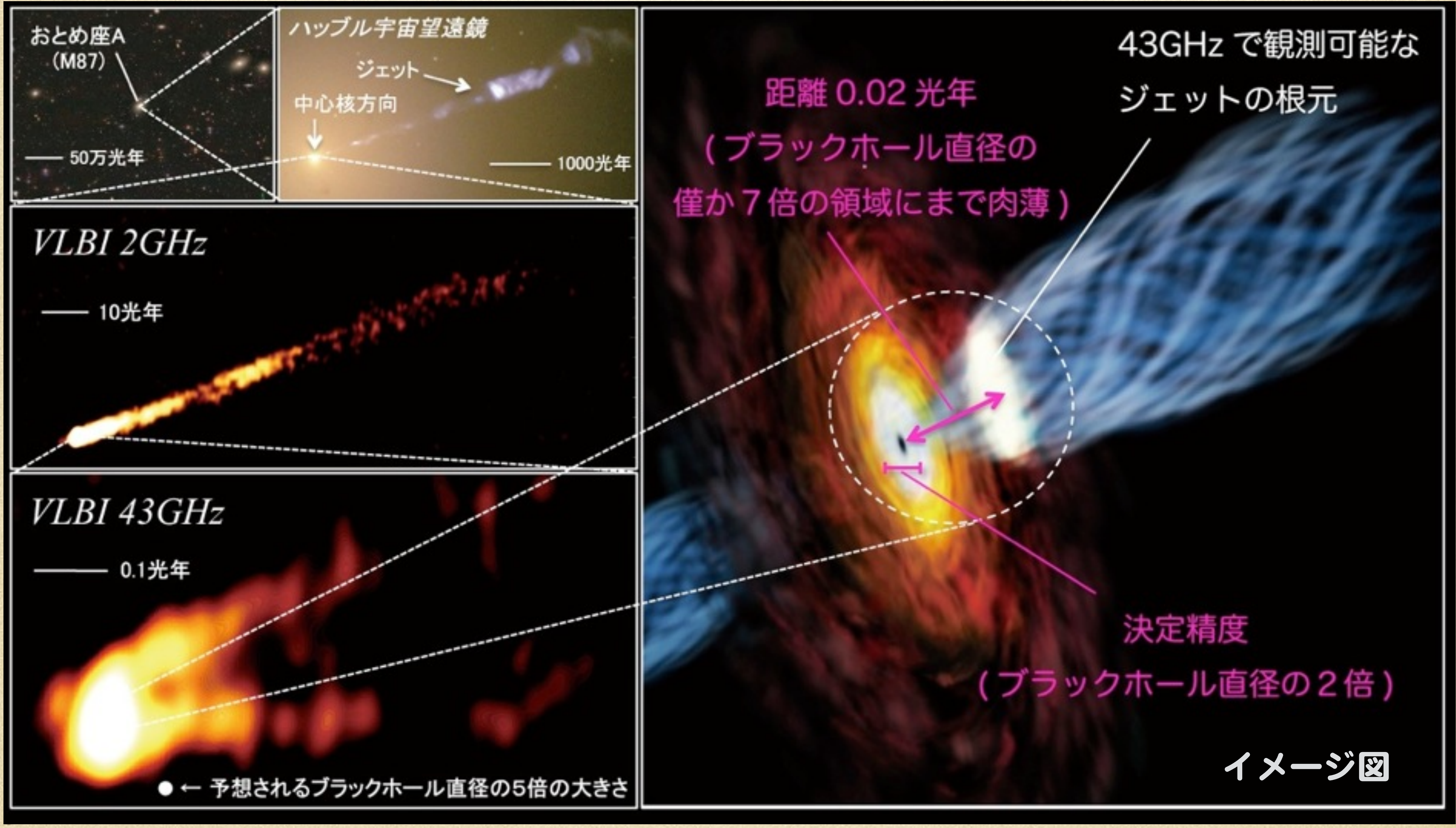
(愛知教育大学)

宇宙ジェットと磁気圏

Where is the black hole power transported to?

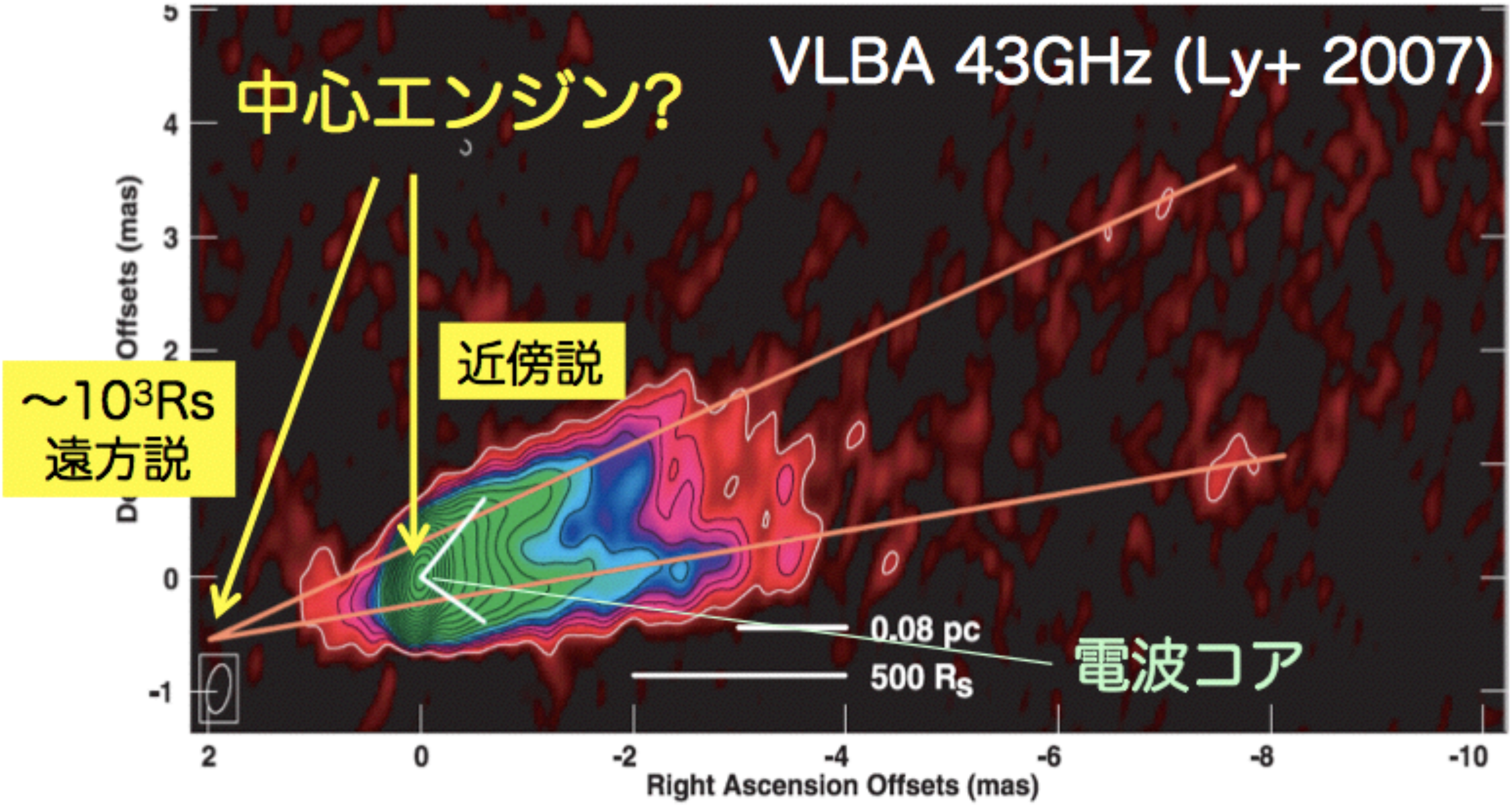
Hada et al. (2011)

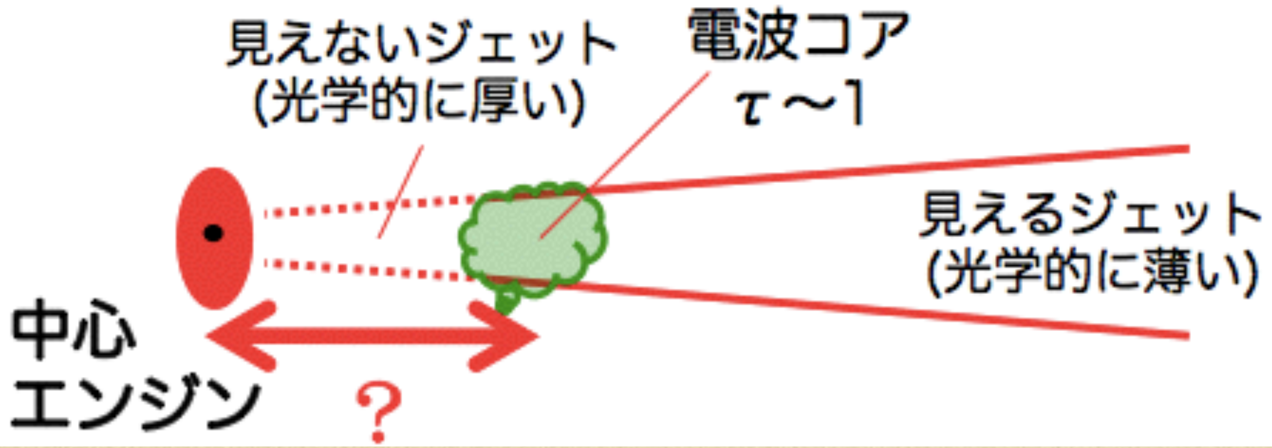
Relativistic Jet : Its origin is still unknown.



（左上段左）Sloan Digital Sky Suvey, (左上段右) NASA and the Hubble Heritage Team, (右)国立天文台/AND You Inc.

中心エンジンは何処に？



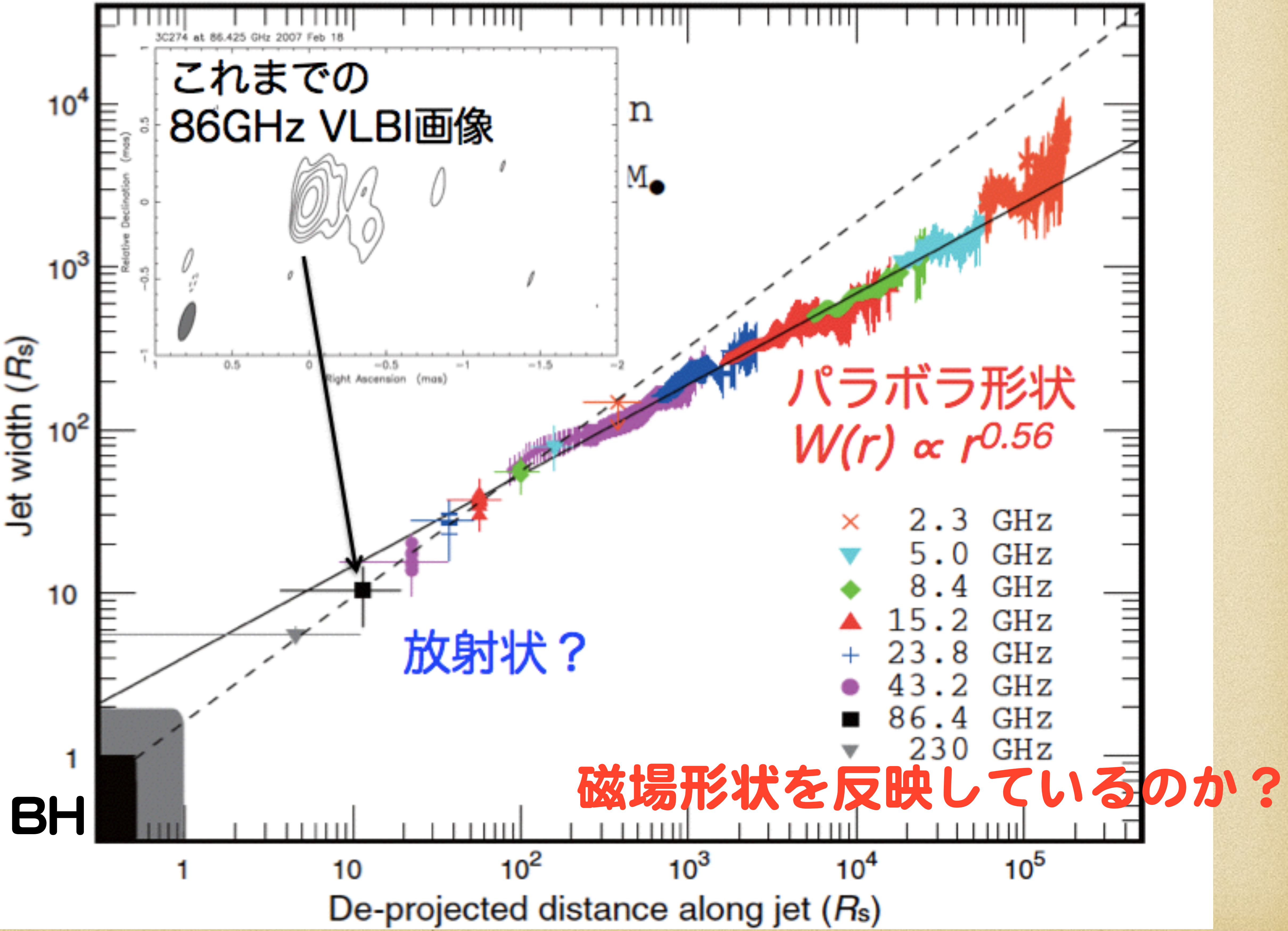


- 電波コア:
  - シンクロトロン自己吸収τ〜1表面

『秦さんプレゼンファイル@銀中祭り2015』より

ジェット収束プロファイル

(Hada et al. 2013, ApJ)



これまでの
86GHz VLBI画像

パラボラ形状
$W(r) \propto r^{0.56}$

放射状？

磁場形状を反映しているのか？

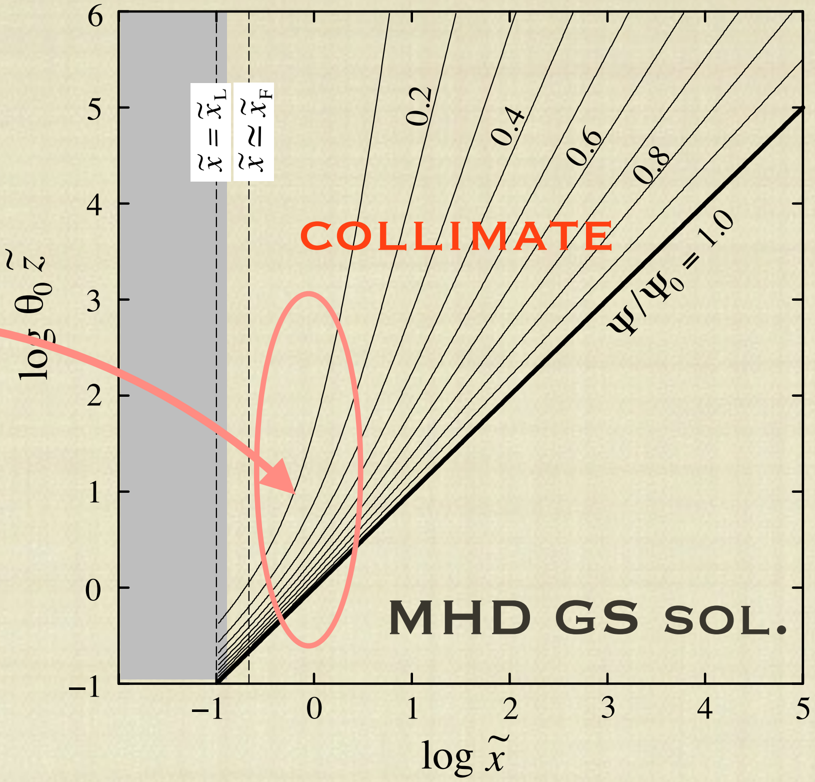
『秦さんプレゼンファイル@銀中祭り2015』より

BH MAGNETOSPHERE: JETS

- **OUTER REGION**

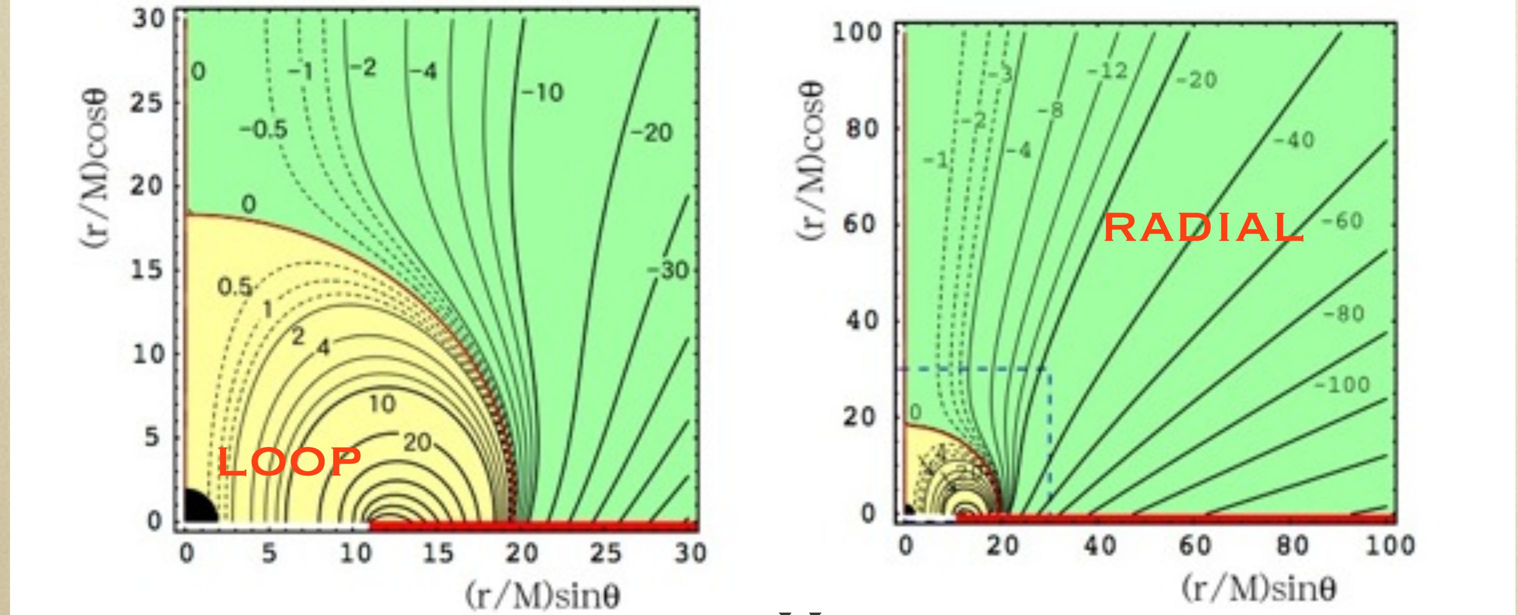
$E_\mathrm{k} \sim E_\mathrm{m}$ の辺りから 折れ曲がる

M87ではどう？



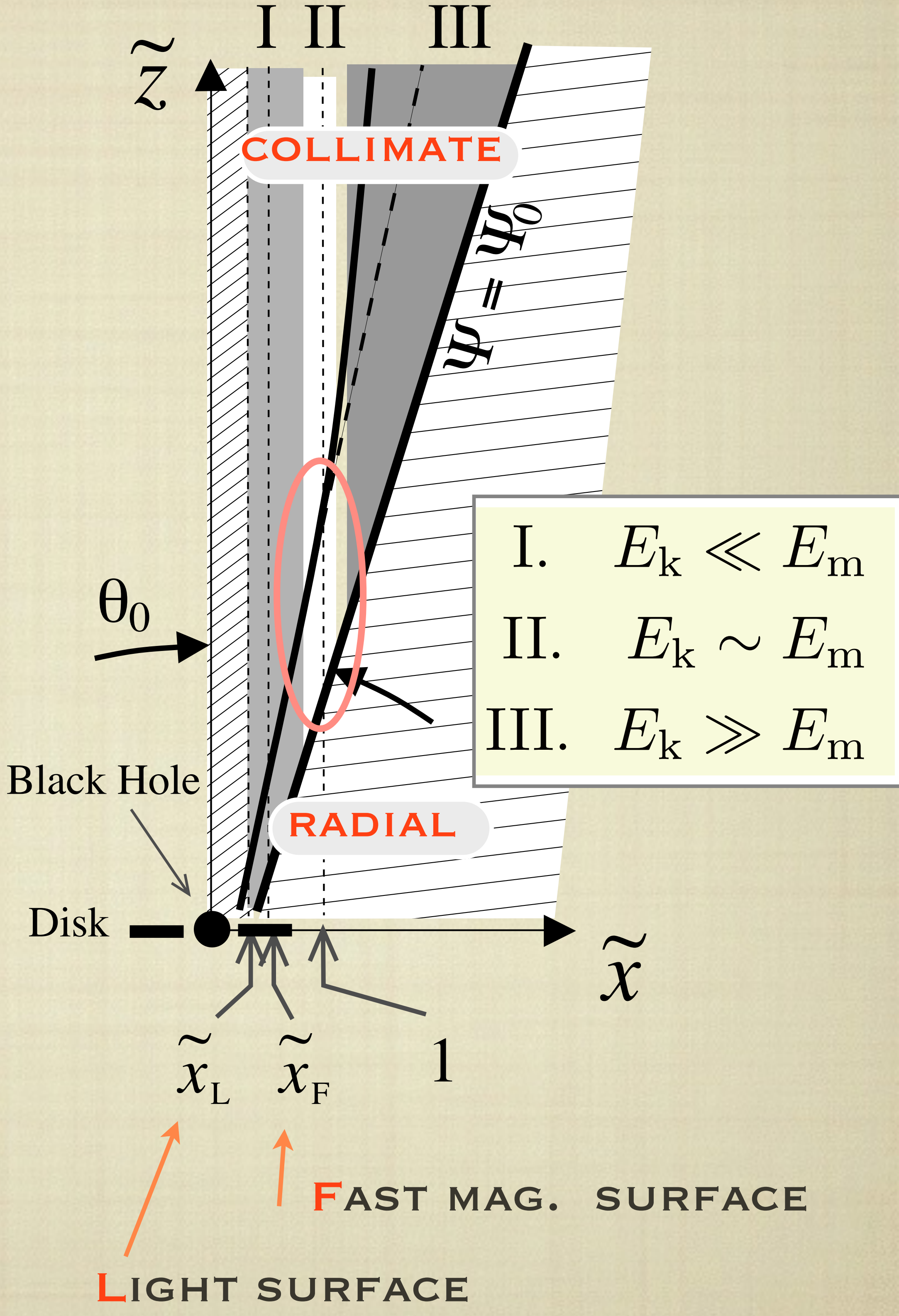
TOMIMATSU & MT 2003

- **INNER REGION**



TOMIMATSU & MT 2000

VACUUM MAXWELL SOL.

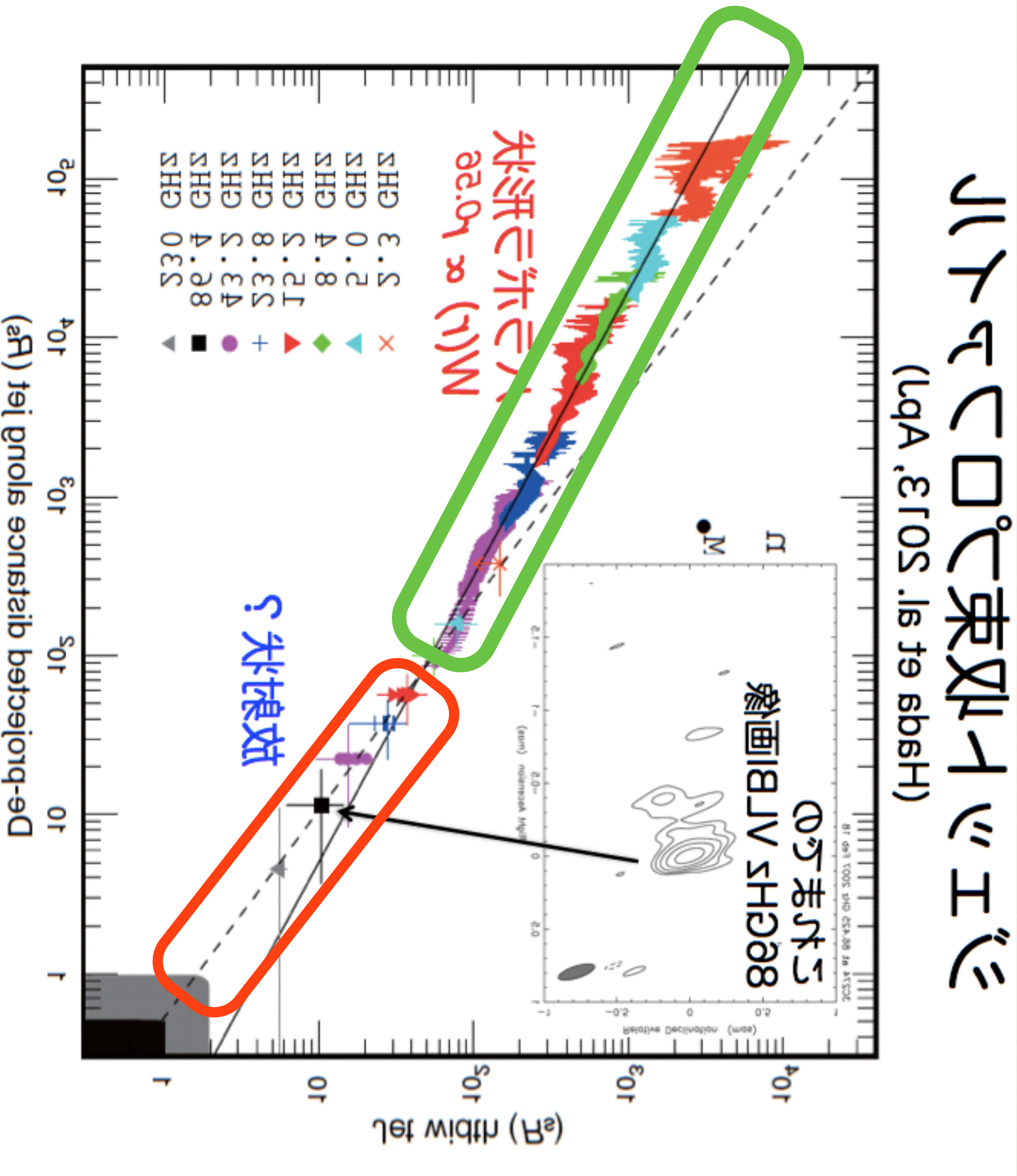


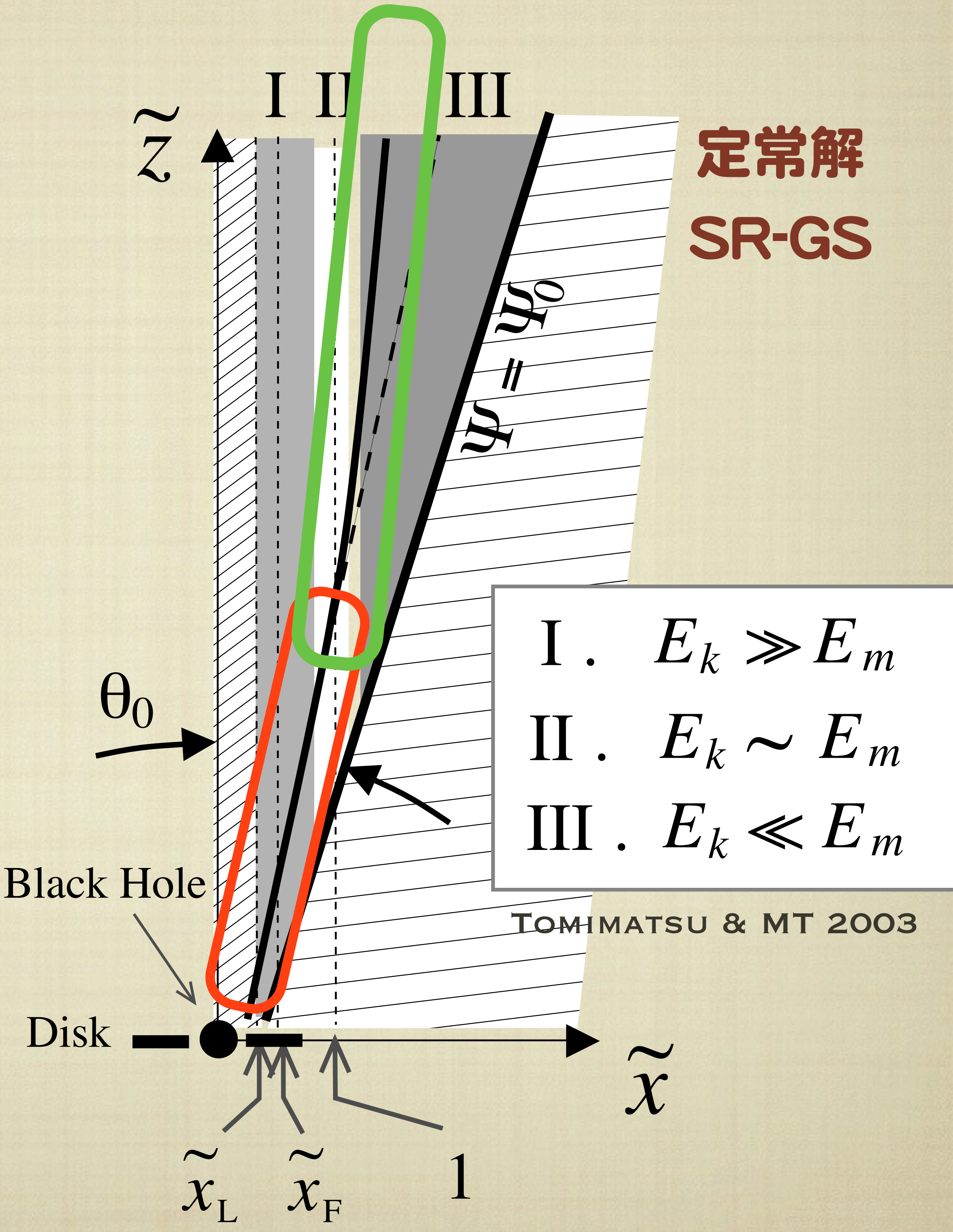
I. $E_\mathrm{k} \ll E_\mathrm{m}$

II. $E_\mathrm{k} \sim E_\mathrm{m}$

III. $E_\mathrm{k} \gg E_\mathrm{m}$

放射状形状　→　パラボラ形状　が見えてきた !?





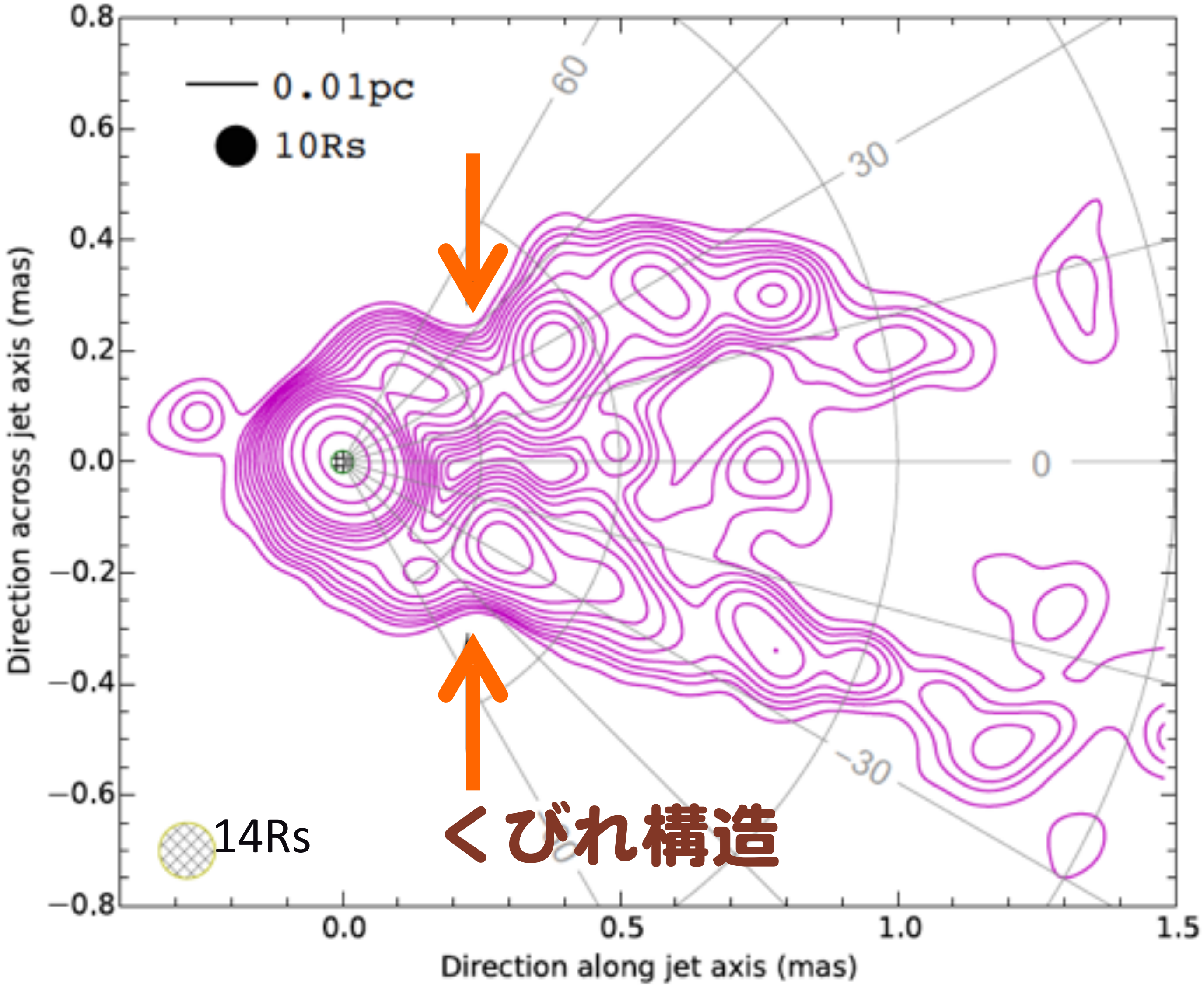
定常解

SR-GS

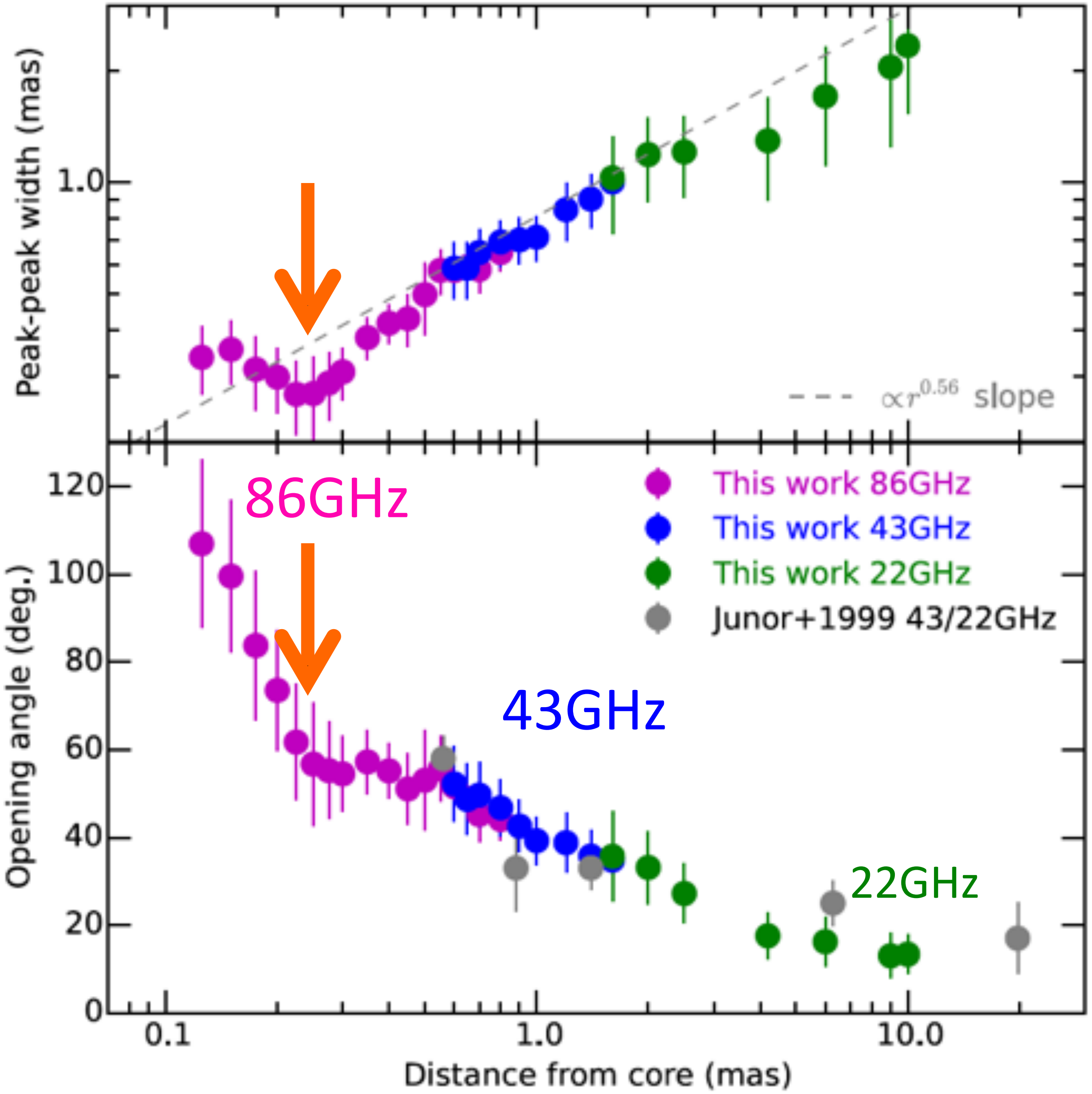
Tomimatsu & MT 2003

r=10-100Rs 生成伝搬 形状

86GHz根元写真 拡大図



ジェット幅(上図) / 開口角(下図)



- 過去の43GHz画像: 開口角~60° at r=100Rs (Junor+1999)
- 今回の86GHz画像: 開口角~100° at r=10Rs
- ピンチ構造@r~30Rs

Hada (2015) in preparation

くびれの原因は？

- **磁気圏の磁場構造？**　　（自発的な構造形成）

  Light surface 以遠では：conical → paraboloidal

  内側領域でのズレは、**強重力場プラズマ環境**に由来？

- **降着円盤内壁を反映？**　　（外因に依る構造形成）

  降着円盤の funnel の形状を反映？

  Funel 壁がノズル状にくびれている？

くびれ位置の時間変化・更なる高分解能像の観測が鍵！

磁気圏とガス円盤は持ちつ持たれつ

- 降着円盤の形成・構造の議論

  円盤表面からのエネルギー・角運動量輸送は構造決定に際して重要。磁場は降着率決定にとって重要な役目を果たす。

- 磁気圏としての取り扱い

  磁気圏の磁場は、主として降着円盤に由来する。磁気圏内のプラズマ流れや磁力線間の力の釣り合いにおいて、降着円盤が境界条件を与える。

相対論的磁気流体力学

GRMHD

GRMHD

General Relativistic Magneto-hydrodynamic

Basic Equations

The *ideal MHD* condition $u^\beta F_{\alpha\beta} = 0$

The particle conservation law $(nu^\alpha)_{;\alpha} = 0$

Maxwell equations $F^{\mu\nu}_{;\nu} = -4\pi j^\mu \ , \quad F_{[\mu\nu;\sigma]} = 0$

Polytropic relation (*Tooper 1965*) $P = K\rho_0^\Gamma$

The equation of motion $T^{\alpha\beta}_{;\beta} = 0$

Field-aligned ``conserved quantities''

flow's parameters

1. Number flux per unit magnetic flux $\eta(\Psi) = \frac{nu^p}{B^p}$
2. Angular velocity of the field lines $\Omega_F(\Psi) = -\frac{F_{tr}}{F_{\phi r}} = -\frac{F_{t\theta}}{F_{\phi\theta}}$
3. Total energy of the magnetized flow $E(\Psi) = \mu u_t - \frac{\Omega_F}{4\pi\eta} B_\phi$
4. Total angular momentum $L(\Psi) = -\mu u_\phi - \frac{1}{4\pi\eta} B_\phi$
5. Entropy $S(\Psi)$

GRMHD Flows

運動方程式を積分して…

- **1. Relativistic Bernoulli equation**

energy in corotation frame

gtavitational Lorentz factor

poloidal magnetic field

$$\boxed{(E - \Omega_F L)}^2 = \mu^2 \alpha + M^2 (\alpha B_p^2 + B_\phi^2)$$

→ PLOT Solution

enthalpy

（rest mass energy＋internal energy）

Alfven Mach number

$$M^2 \equiv \frac{u_p^2}{u_{\rm AW}^2}\alpha$$

toroidal magnetic field

- **2. total Energy of MHD Flow**

時間成分を積分して…

$$E(\Psi) \; = \; \mu u_t \; \boxed{- \; \frac{\Omega_F}{4\pi\eta} B_\phi}$$

energy conversion
KE <--> ME

the fluid part of energy

Poynting flux per the particle number flux

Bernoulli equation (Newtonian)

- From equation of motion,

gravitational potential

Poynting flux

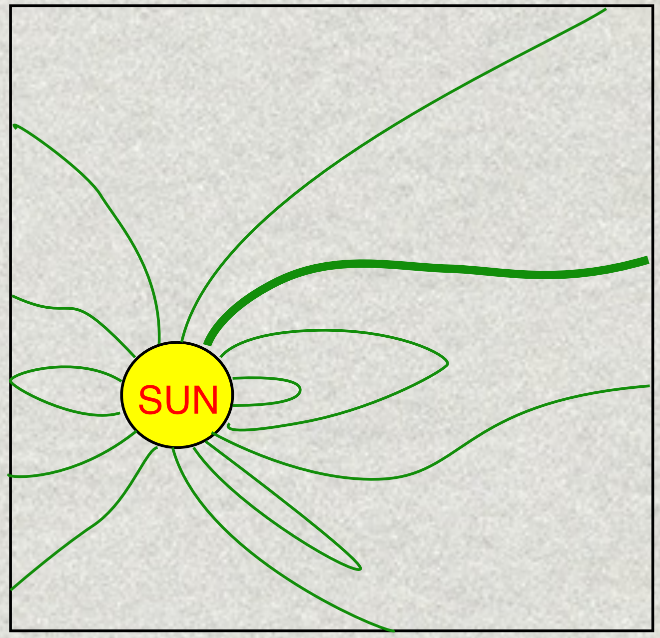
$$E = \left(\frac{1}{2}v_r^2 + \frac{1}{2}v_\phi^2 + \frac{\Gamma}{\Gamma - 1}\frac{P}{\rho} + \Psi(r)\right) - \frac{rB_\phi B_r \Omega_F}{4\pi\rho v_r}$$

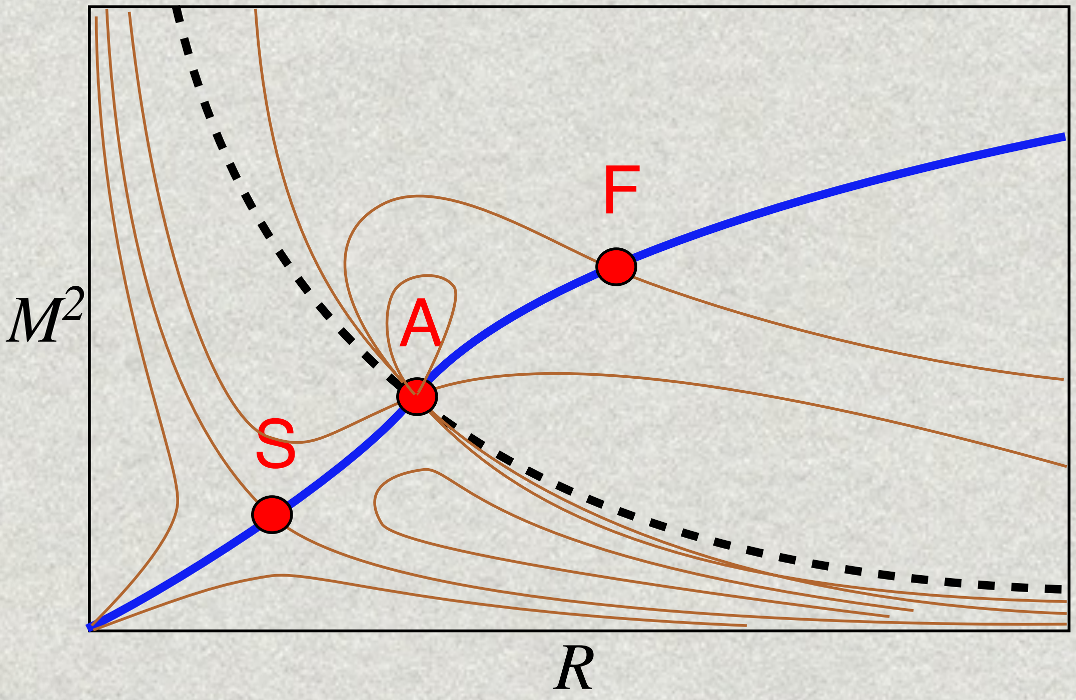
kinetic energy internal energy

OR

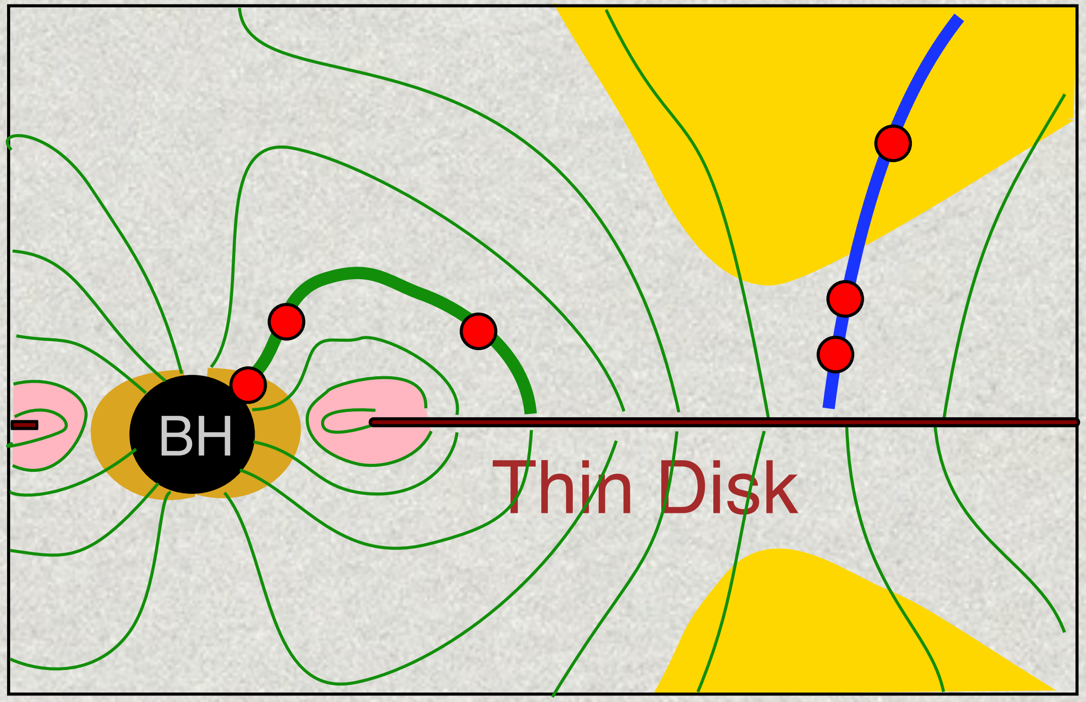
$$(E - \Omega_F L) = \frac{1}{2}v_r^2 + \frac{1}{2}v_\phi^2 + \frac{a_{\rm sw}^2}{\Gamma - 1} + \Psi(r) - rv_\phi\Omega_F$$

SOLAR WIND





ブラックホール磁気圏

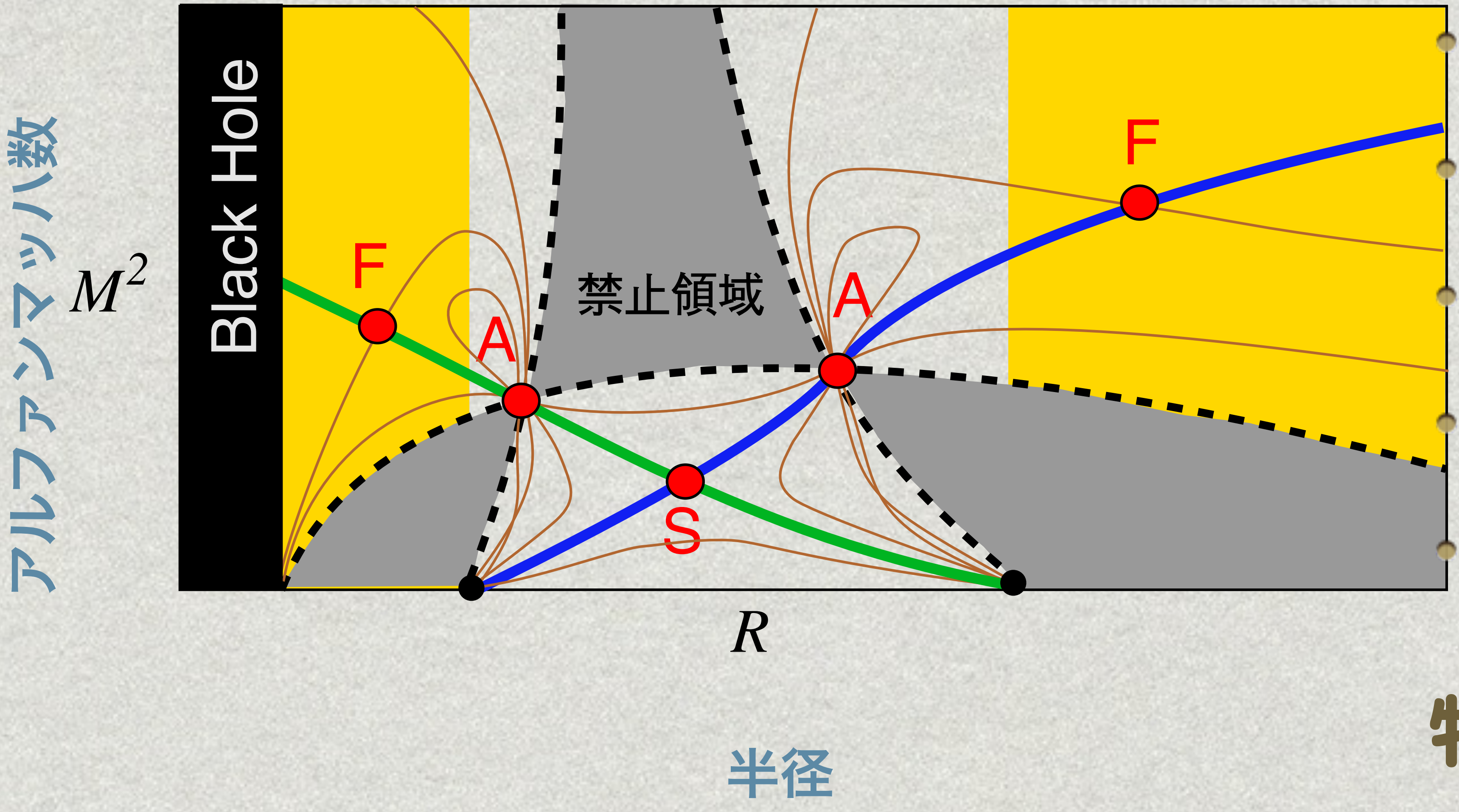


INFLOW と OUTFLOW が共存

相対論効果：

LIGHT SURFACE

禁止領域（$E^2 < 0$）



- プラズマ源
- 遅い磁気音速点
- アルフェン点
- 速い磁気音速点
- ブラックホール地平面

特徴的な半径たち

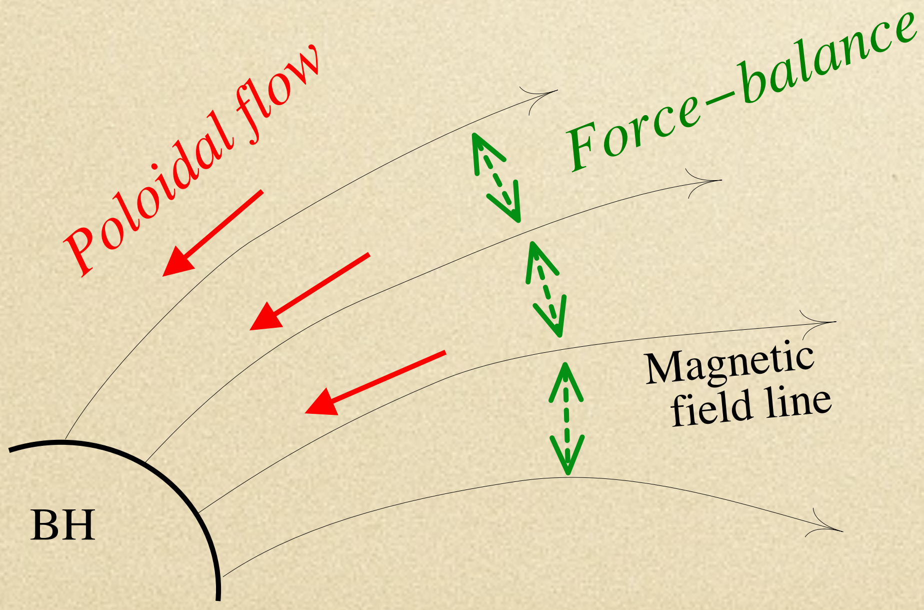
3. Trans-field equation

The force-balance equation is derived by Nitta et al.(1991)

GRMHD

$$\begin{aligned}
&\frac{\alpha - M^2}{4\pi}(\partial^r\partial_r\Psi + \partial^\theta\partial_\theta\Psi) + \frac{\hat{B}_p^2\rho_w^2}{4\pi\sqrt{-g}}\left[\frac{\sqrt{-g}}{\rho_w^2}(\alpha - M^2)\right]' \\
+\ & \frac{4\pi\mu^2\rho_w^2}{M^2}\eta\eta' + \frac{2\pi}{M^2}\left[g_{\phi\phi}(E^2\eta^2)' + 2g_{t\phi}(EL\eta^2)' + g_{tt}(L^2\eta^2)'\right] \\
-\ & \frac{4\pi\eta^2\rho_w^2}{M^4}(\mu^2 G_\phi - e\mu u_\phi)\Omega_F' + \frac{4\pi}{M^2}\left[G_\phi(E\eta)' + G_t(L\eta)'\right](E\eta)\hat{f} \ = \ 0
\end{aligned} \tag{7.47}$$

where the prime $(') \equiv -(1/\hat{B}_p^2)[(\partial^r\Psi)\partial_r + (\partial^\theta\Psi)\partial_\theta]$, $\hat{B}_p \equiv \rho_w B_p$ and $\hat{f} \equiv \rho_w f$.



The ideal MHD plasmas stream along magnetic field lines.

The configuration of field lines is determined by the force–balance between the field.

Inflow & Outflow

GRMHD

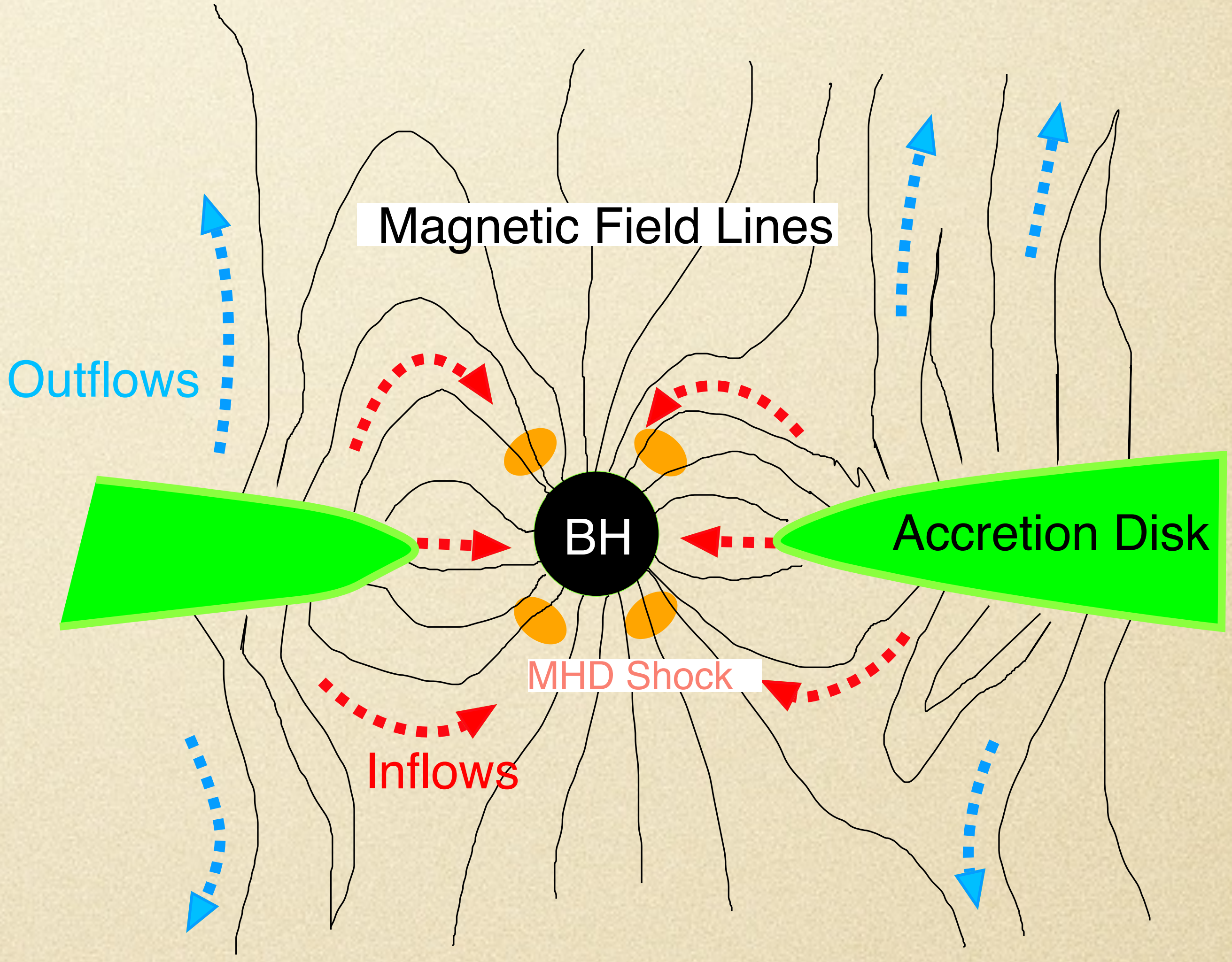
Inflow and Outflow

- ブラックホールの周りにプラズマ源が必要！

  降着ガス円盤
  コロナ

- 降着流＋ジェット／ディスク風

- ２つの磁気圏領域をプラズマ源で接続



磁気圏は？ --- 磁気流体力学(GRMHD) を適用するの解析

＊ Kerr時空，定常･軸対称，理想MHD，磁場形状，を仮定

＊ 磁気流体の保存量，BH spin, 磁場分布，がパラメータ

$$E,\ L,\ \Omega_F,\ \eta,\ \mu_{\rm hot},\ a,\ B_p(r,\theta)$$

＊ 磁気音速点を通過するための臨界条件がある

β-model (Tomimatsu & M.T. 2003, M.T & Tomimatsu 2008)

ポロイダル磁場とトロイダル磁場の比をモデル化する

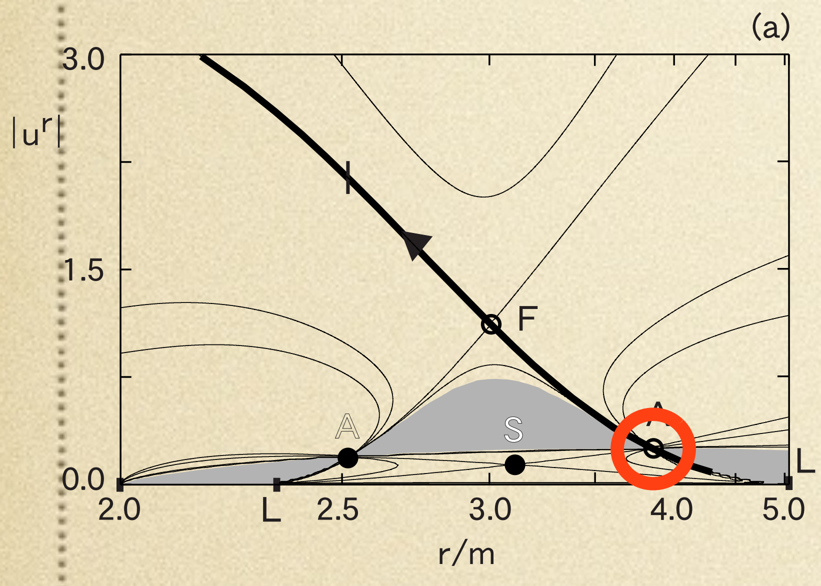
＊ 磁力線の間の力の釣り合いを研究する時代に

＊ プラズマ源の物理の開拓を

MHD Accretion onto Black Holes

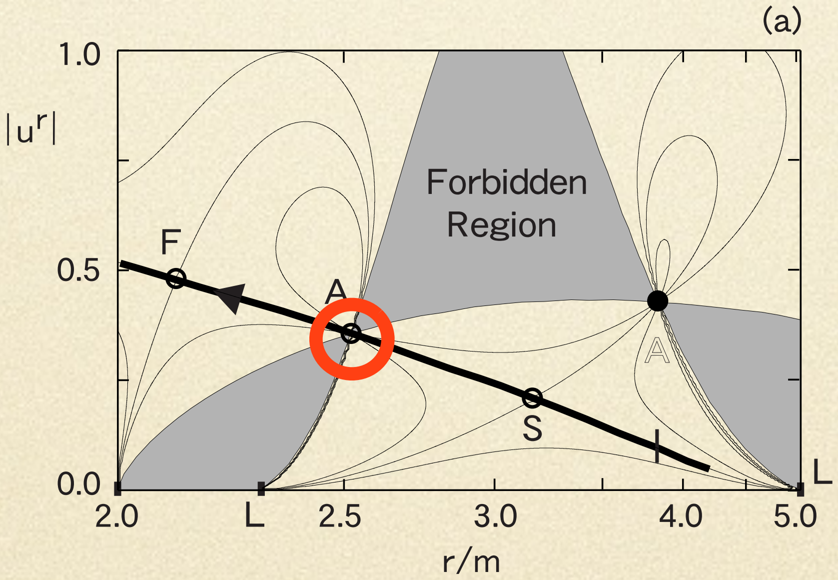
２つのアルフェン点　→　磁気的／流体的降着流

BH: slowly rotating



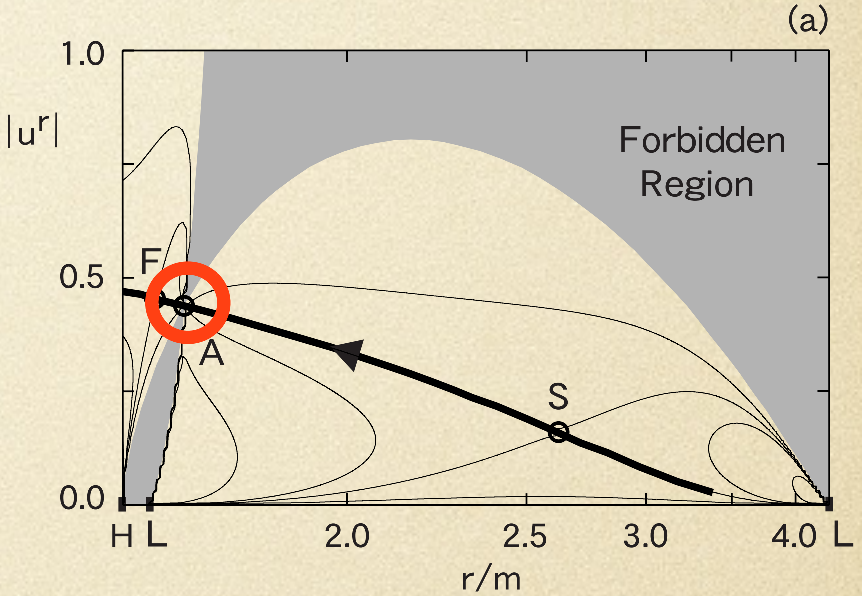
hydro-like

BH: slowly rotating



magneto-like

BH: rapidly rotating



magneto-like

weak-magnetic field limit

=> Hydro Dynamical Flow

strong-magnetic field limit

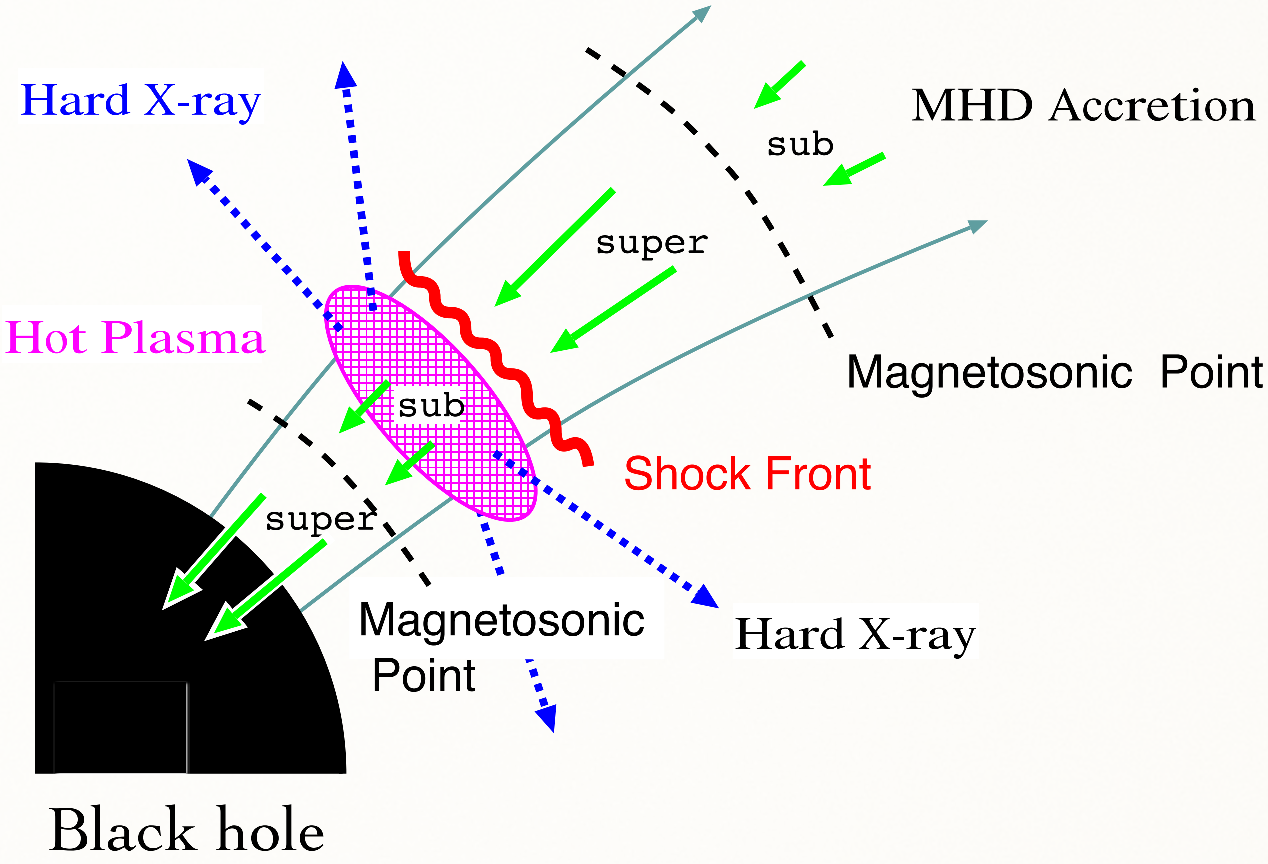
=> Force-free magnetosphere

Some critical points make regularity conditions.

Plasma source => Slow Point => Alfven P. => Fast P. => Event Horizon

(boundary conditions)

衝撃波で加熱：BH近傍からの放射（観測量となる）



Plasma Source
(accretion disk / corona)
↓
Slow magnetosonic point
↓
Alfven point
↓
Fast magnetosonic point → MHD Shock
shock-free flow ↓ shocked flow ↓
Event Horizon ← 2nd *Fast magnetosonic point*

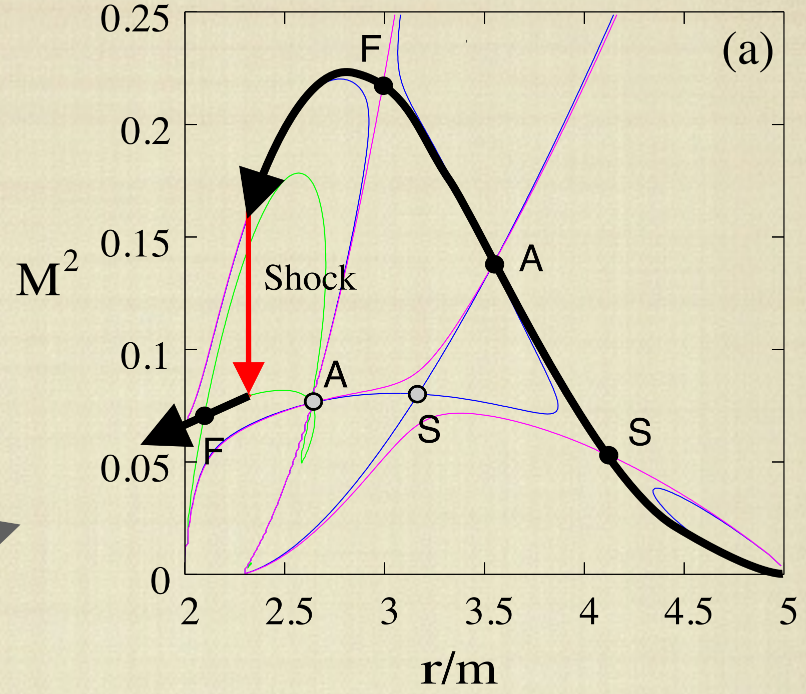
MHD Accretion onto BH

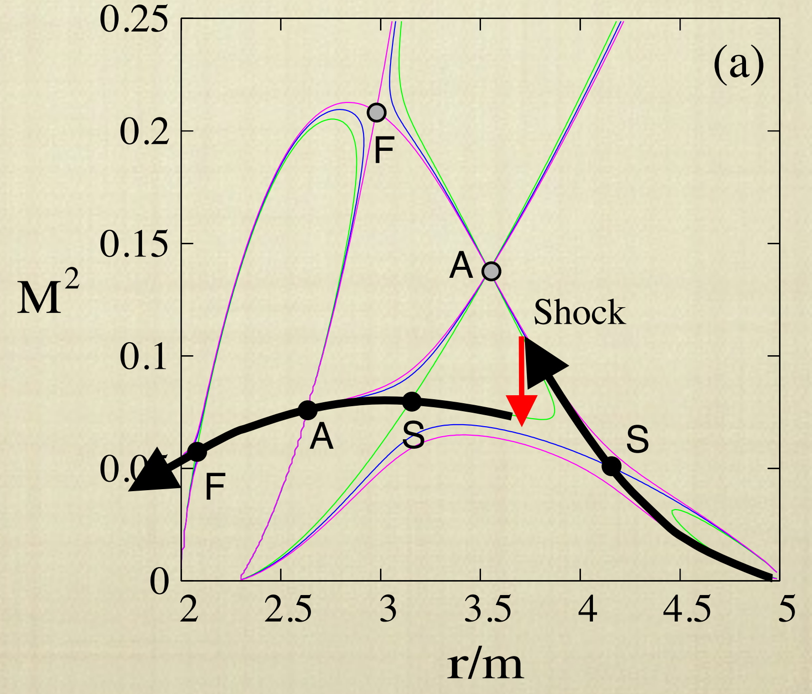
Hot Plasma region

for high energy radiation

- Fast magnetosonic Shock
  - Hydro-like
  - Magneto-like
- Slow magnetosonoc Shock

Takahashi et al. 2006



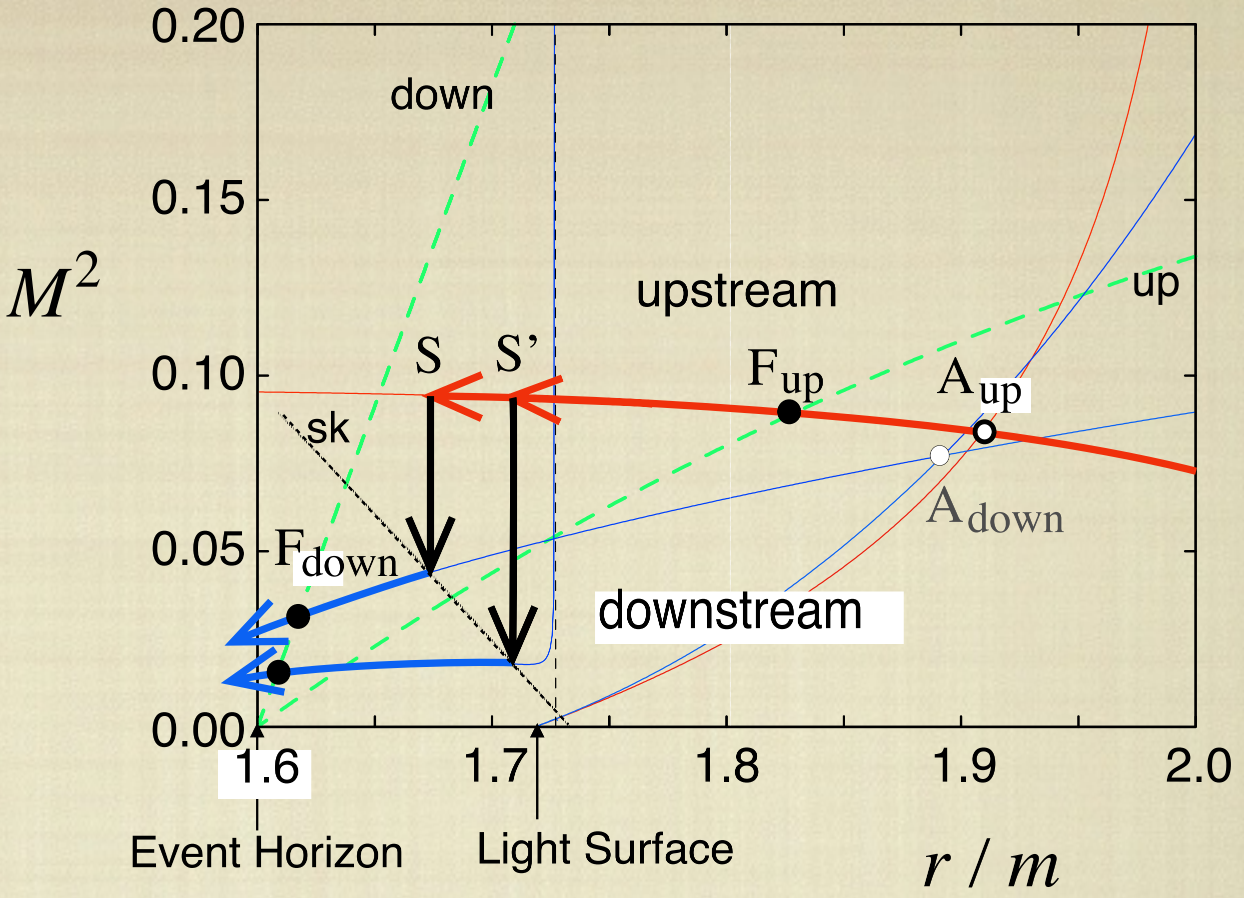


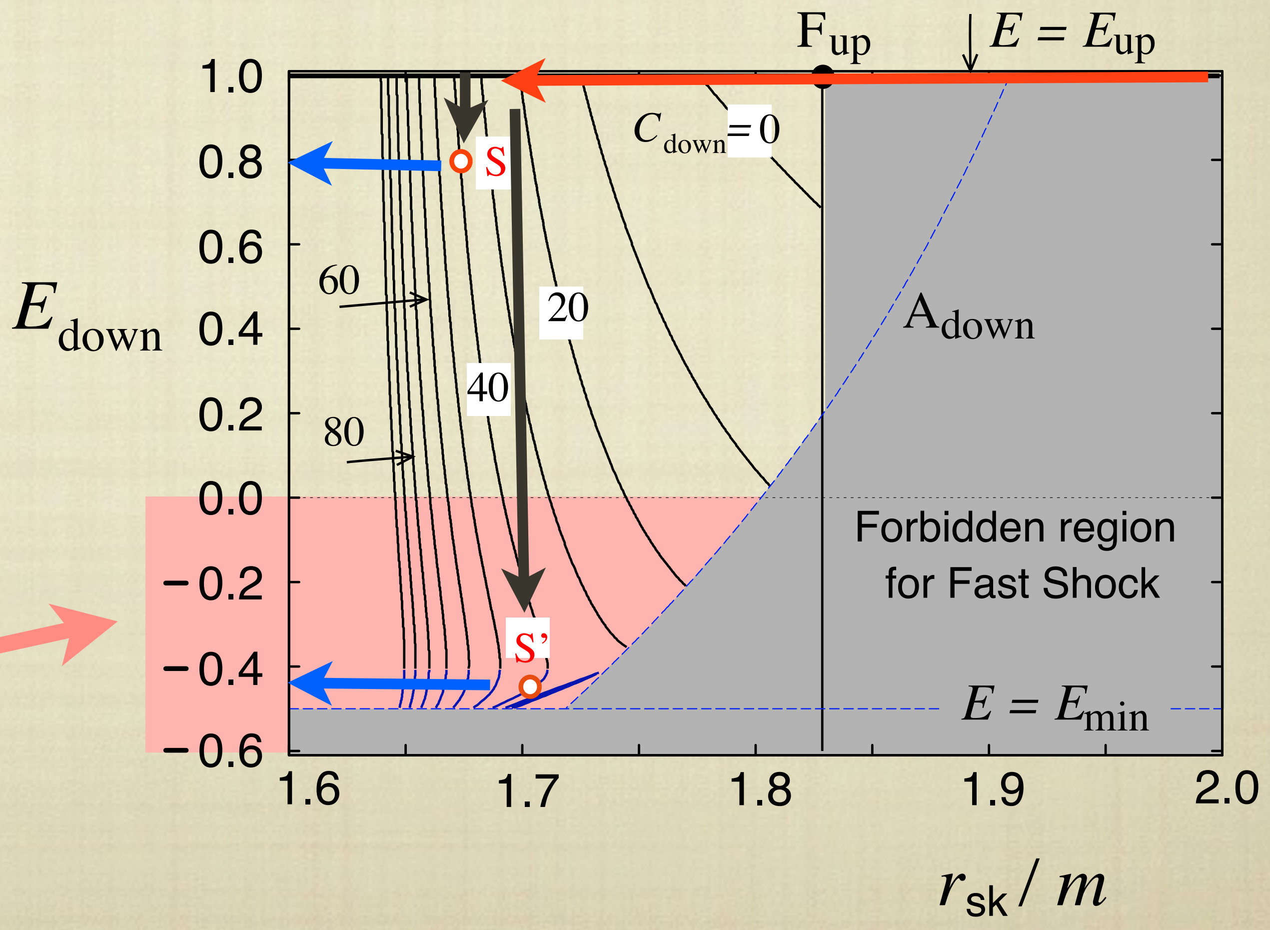
Black Hole MHD Shock

- Fast magnetosonic shock in ergosphere
- cold upstream → cold downstream
- S' : $\Delta E > mc^2$

Negative Energy inflow

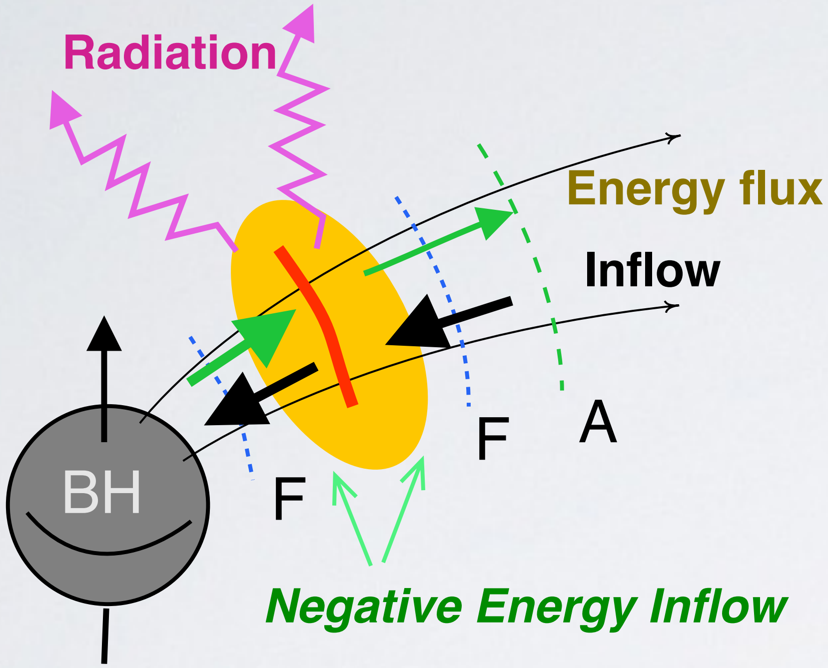
MT & Takahashi . 2010

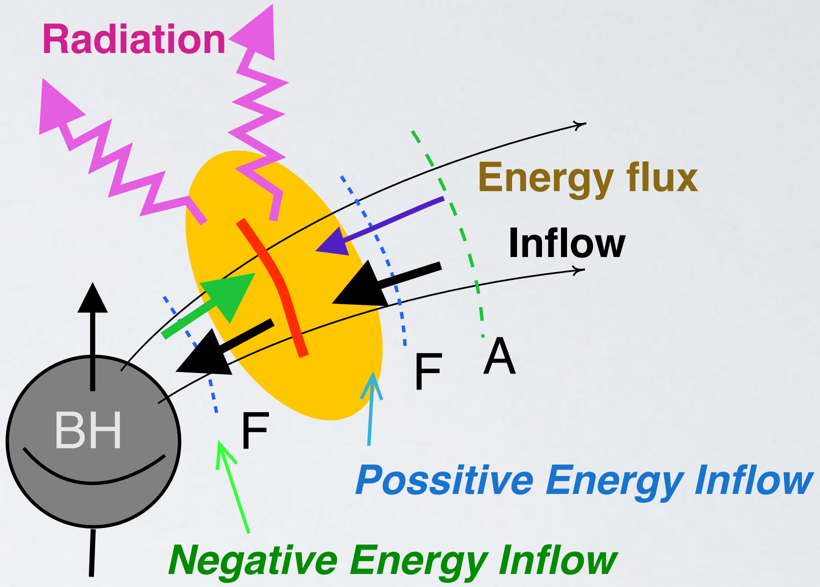


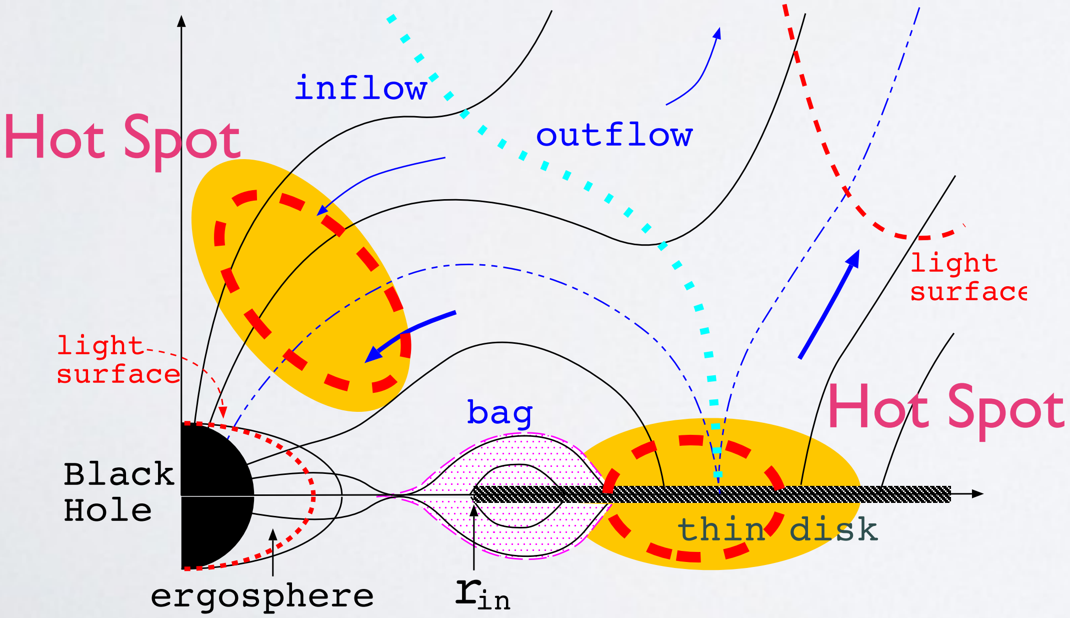


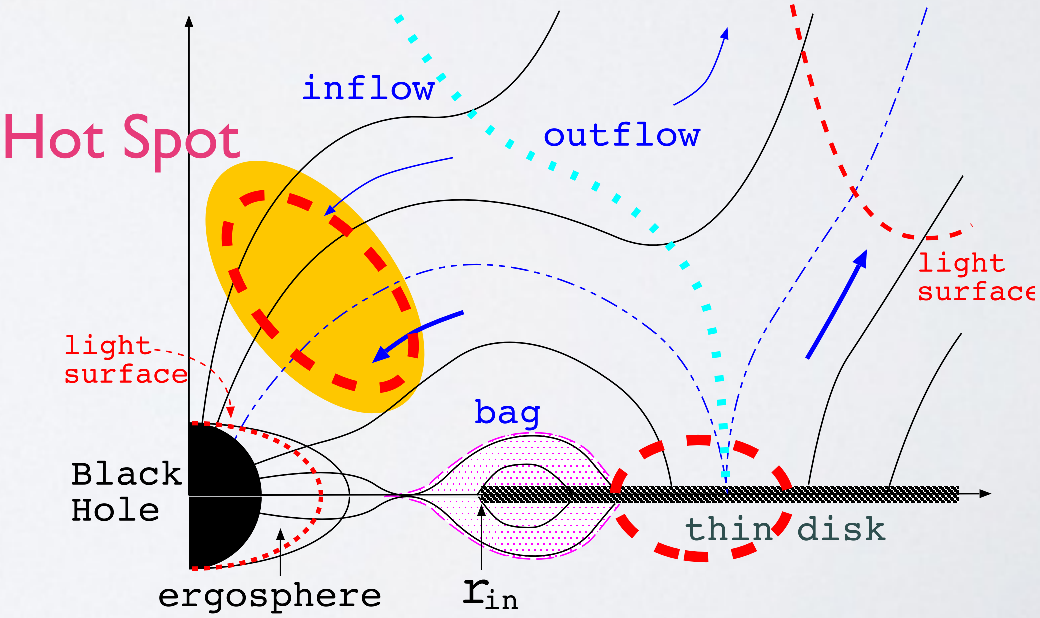
Hot Spot powered by Rotating BH

高エネルギー放射　＝　観測できる !?　　鉄輝線の光源？





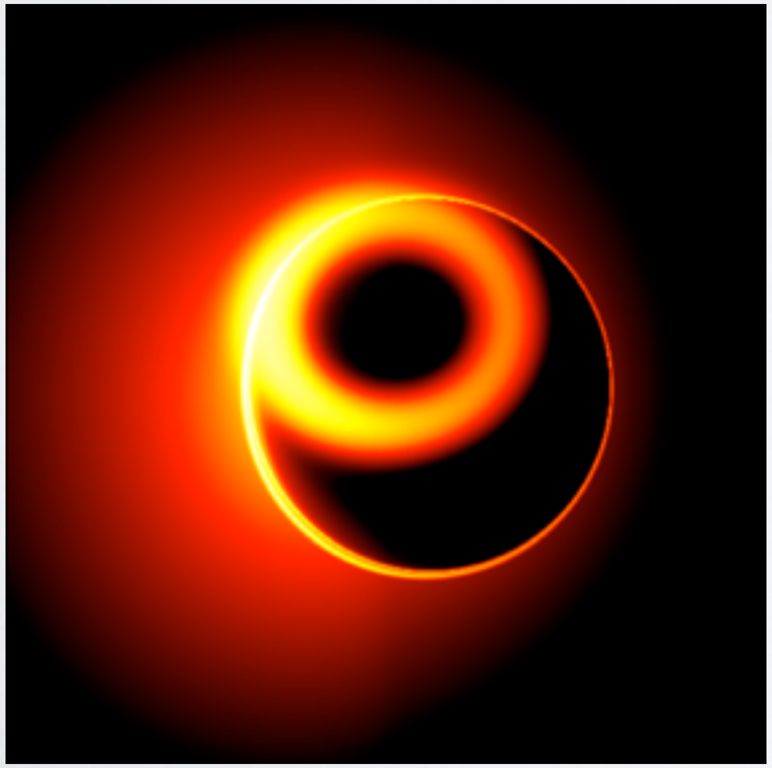


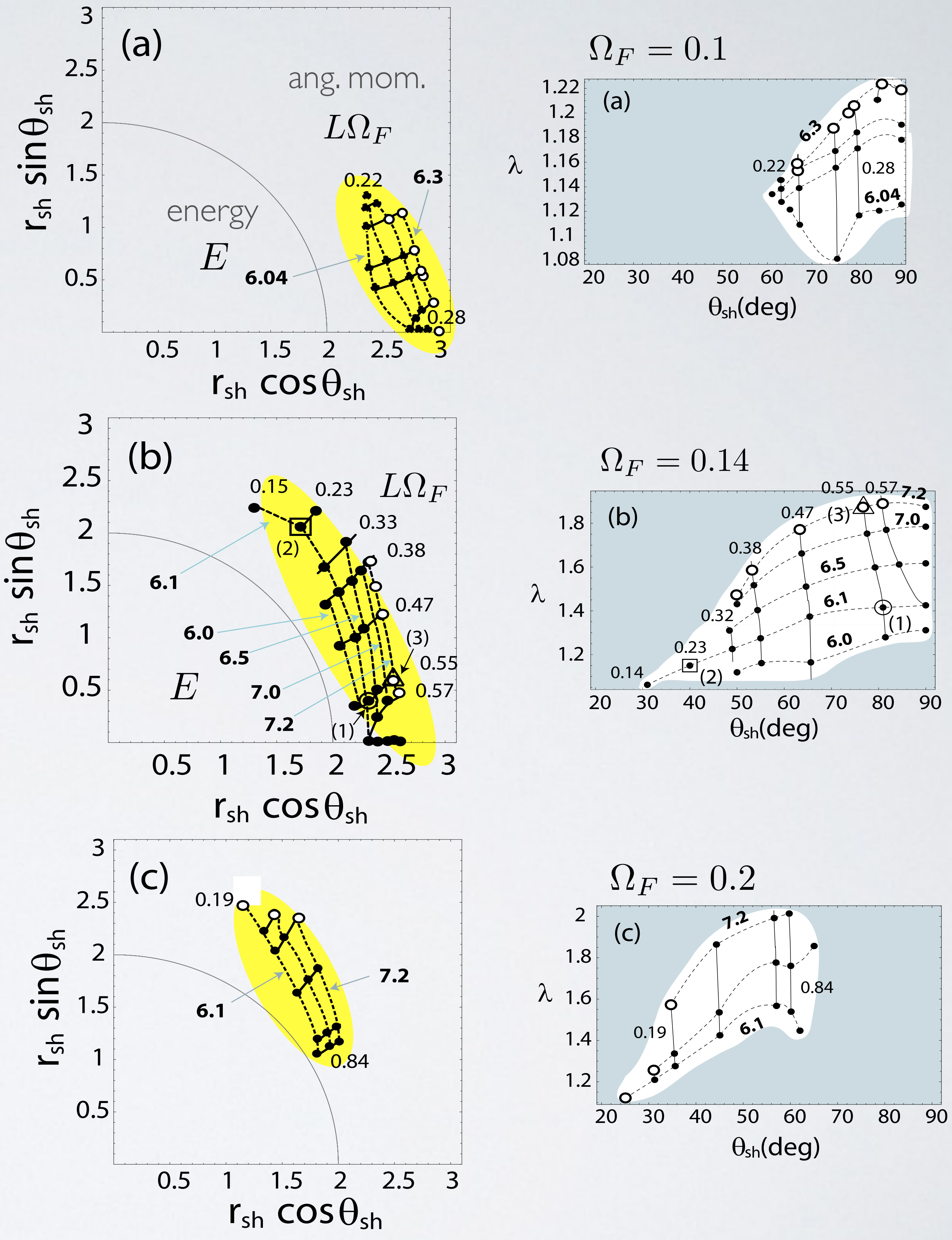


MHD Shock : Adiabatic cases (2)

Takahashi et al. (2006)
Fukumura et al. (2007)

for example , , , ,





Acceptable shock locations

Black Hole Magnetosphere

GRMHD　定常解

ブラックホール磁気圏モデル

磁場の源：　ring-current, disk-current

- **真空磁気圏**

  No current in the magnetosphere

- **Force-free 磁気圏**

  Magnetically-dominated magnetosphere

  電流は磁力線に沿って流れる＝力なし

- **MHD磁気圏**

  Magnetohydrodynamical Plasma

  磁力線を横切る電流が存在＝プラズマ加速

MHD 磁気圏の解のために

- 流線に沿って５つの保存量が存在
- 磁力線に沿う方向の運動方程式
- 磁力線に垂直方向の力の釣り合いの式

降着ガス円盤

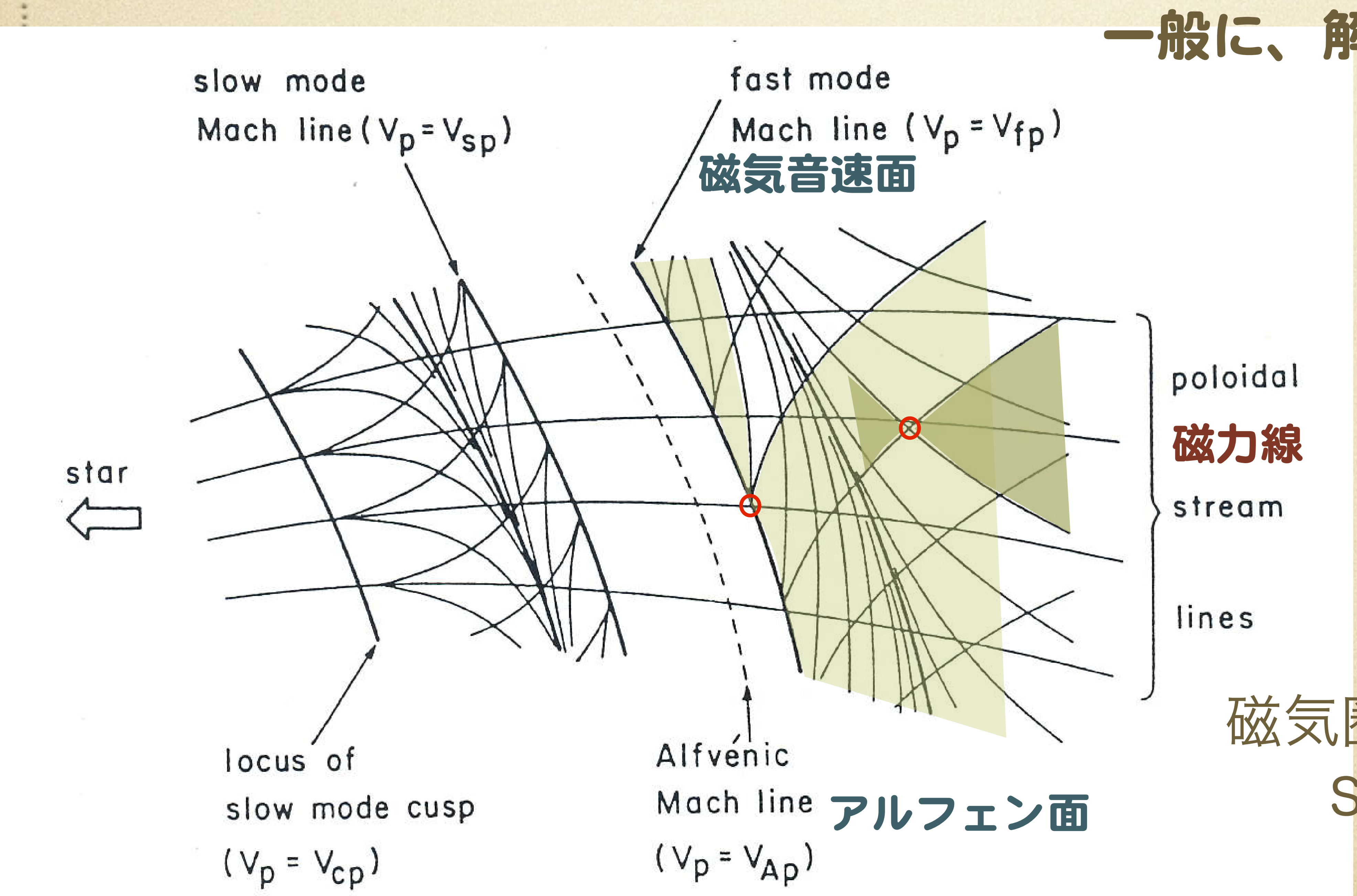
適切な境界条件@プラズマ源を考察して、

上記を連立させて解く

MHD磁気圏　定常解

- 楕円型と双曲型のハイブリッド方程式
- 境界値問題と波動方程式をイタレーティブに解く

一般に、解くのは困難！



磁気圏の因果関係
Sakurai 1990

BZ Flux

BLANDFORD & ZNAJEK (1977)

(1) Poynting-flux (r-compornant):

$$\mathcal{E}^r(r,\theta) \equiv T^r{}_t = \epsilon_0 \frac{E_\theta B_\phi}{\sqrt{-g}}$$

where $\sqrt{-g} = \Sigma \sin\theta$, $\Sigma = r^2 + a^2 \cos^2\theta$ and $E_\theta = \Omega_F F_{\theta\phi} = \Omega_F A_{\phi,\theta}$.

(2) The boundary condition at the horizon : toroidal compornent of magnetic fields

$$B_\phi^H[\Psi(\theta)] = (\omega_H - \Omega_F)\frac{(r_H^2 + a^2)\sin\theta}{\Sigma_H}(A_{\phi,\theta})_H$$

(3) Blandford-Znajek Power (Poynting-flux at the horizon):

$$\mathcal{E}_H^r[\Psi(\theta)] = \epsilon_0 \boxed{\Omega_F(\omega_H - \Omega_F)} \frac{(r_H^2 + a^2)}{\Sigma_H^2} \boxed{(A_{\phi,\theta})_H^2}$$

BZ PROCESS

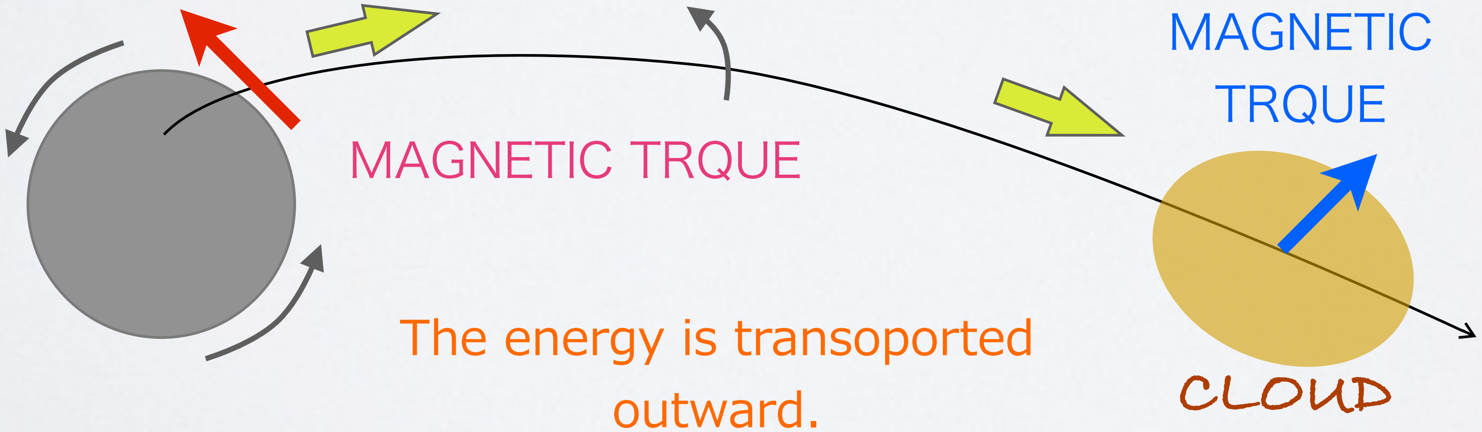
BLANDFORD & ZNAJEK (1977)

☐ MAGNETIC TRQUE

$$T_\phi = \frac{1}{4\pi} B_p B_\phi \sim -\frac{(\omega - \Omega_F)}{4\pi\sqrt{-g}} \frac{(A_{\phi,\theta})^2}{\sqrt{\Sigma}}$$

☐ ANGULAR MOMENTUM / ENERGY FLUX (WHEN $B_\phi < 0$ ---> OUTWARD)

$$\mathcal{L}^r = -\sqrt{g_{\phi\phi}}\, T_\phi \qquad \mathcal{E}^r = \Omega_F \mathcal{L}^r$$



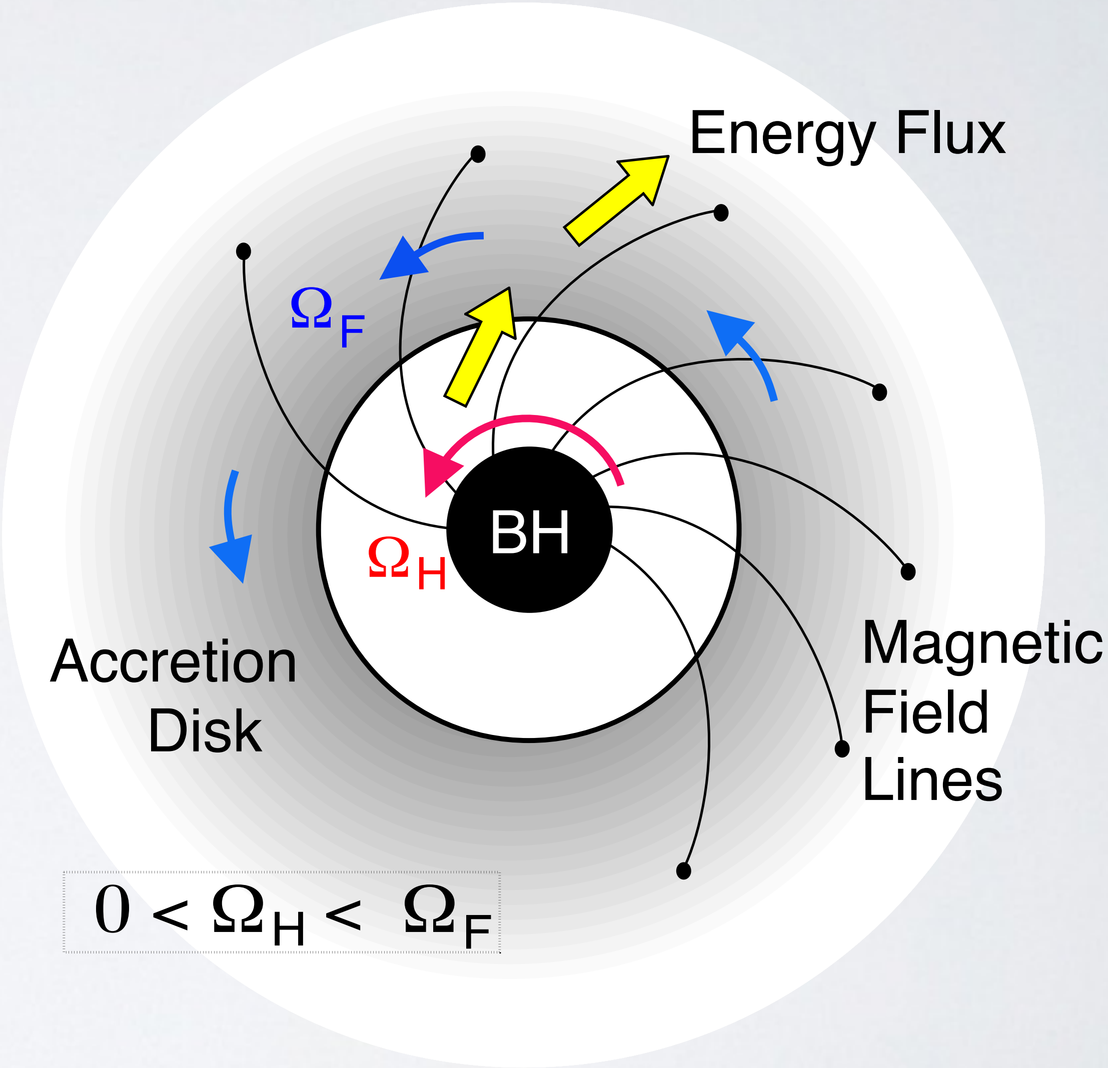
The energy is transosported outward.

When the Alfven point locates inside the Ergosphere,

Energy Extraction from Rotating BH by MHD Inflows is possible.

$$E(\Psi) = \mu u_t - \frac{\Omega_F}{4\pi\eta} B_\phi$$

	fluid part	EM part
$E(\Psi)$	$= \mu u_t$	$- \frac{\Omega_F}{4\pi\eta} B_\phi$
$-$	$+/-$	$-$
Negative energy inflow	gas accretion	BZ process



This situation would be possible in the BH magnetosphere

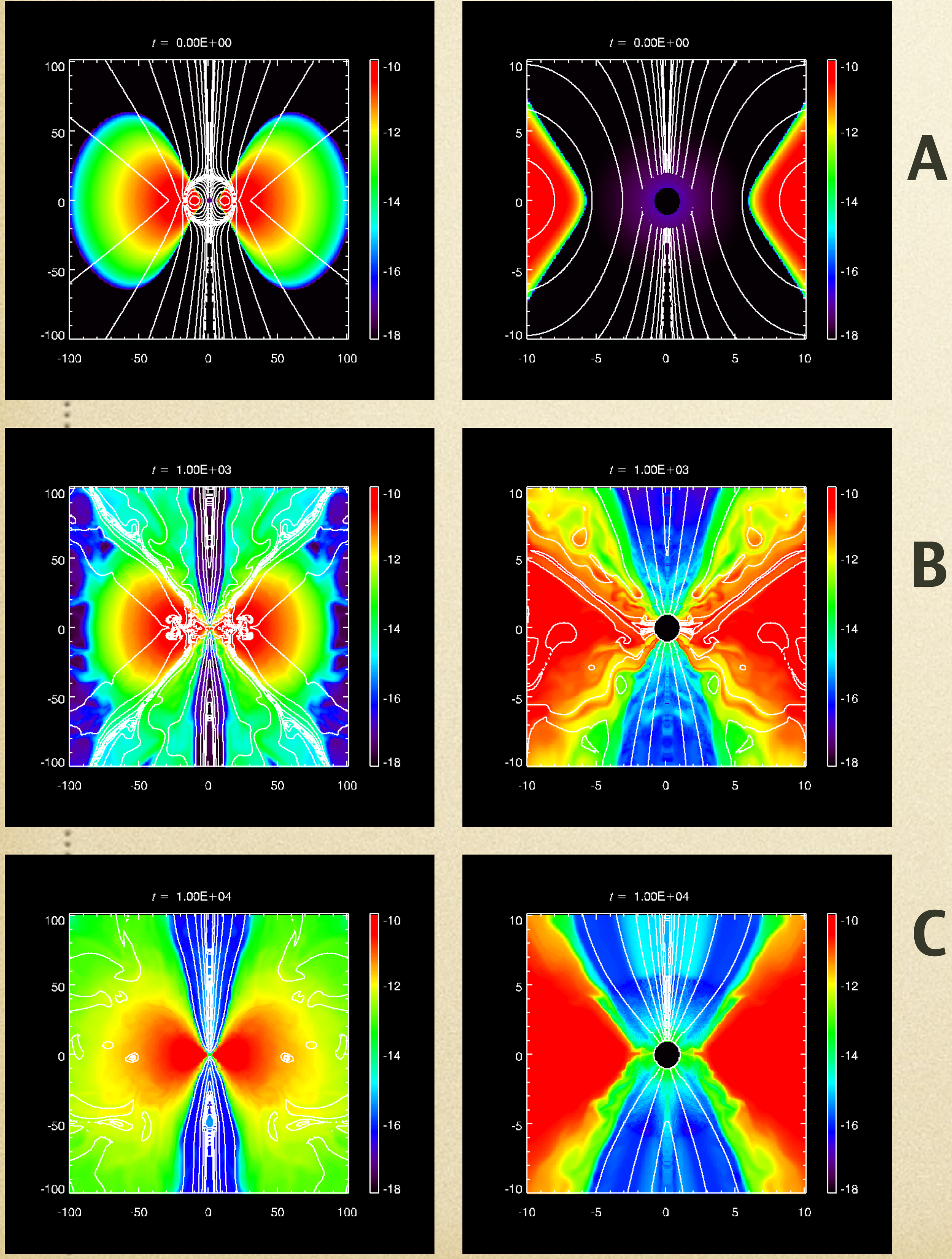
(magnetically dominated off-equatorial region).

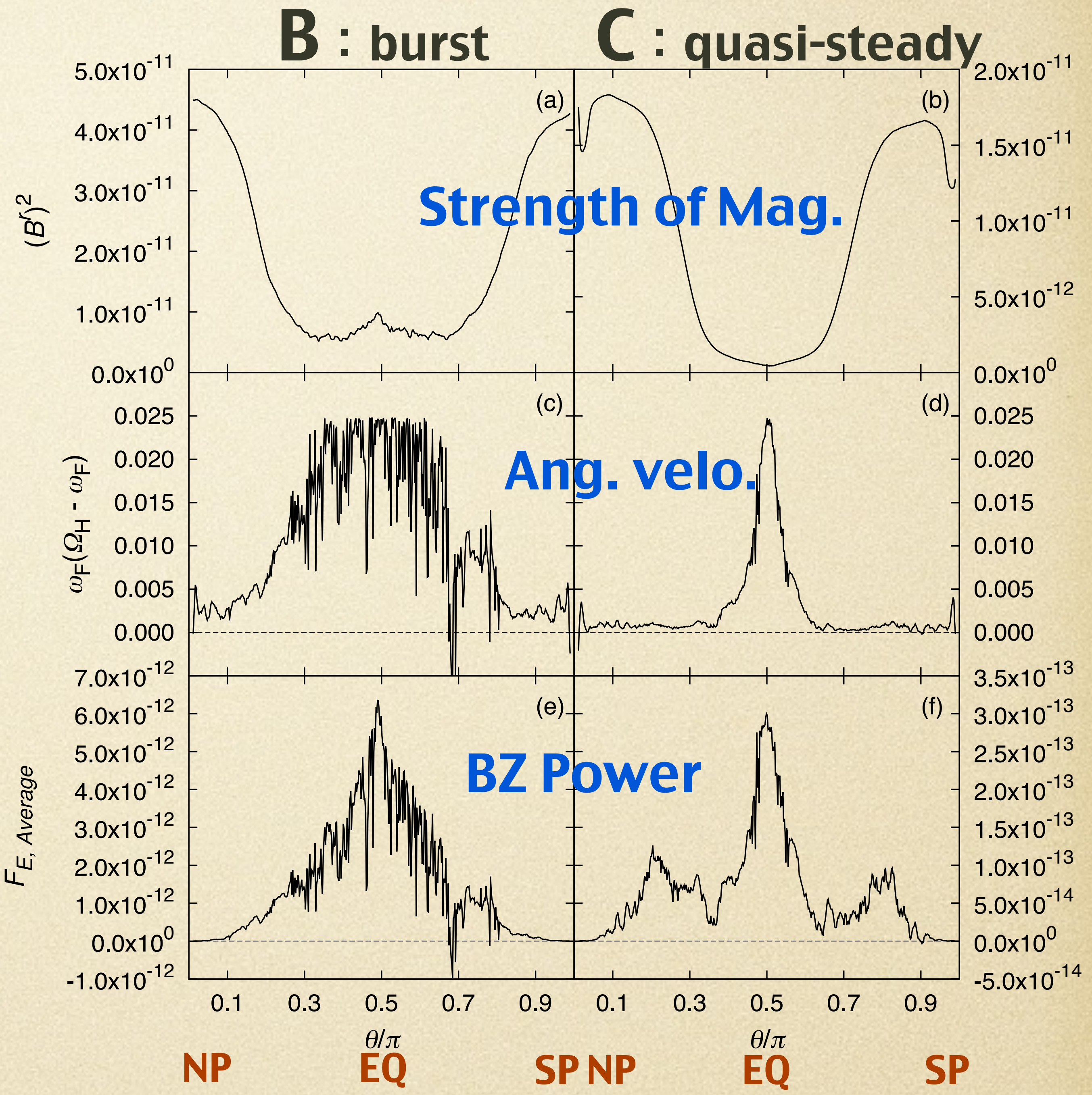
(see M. Takahashi, et al. 1990)

GRMHD Simulation

Maeda+ 2014

BH+Acc.Disk+Magnetosphere



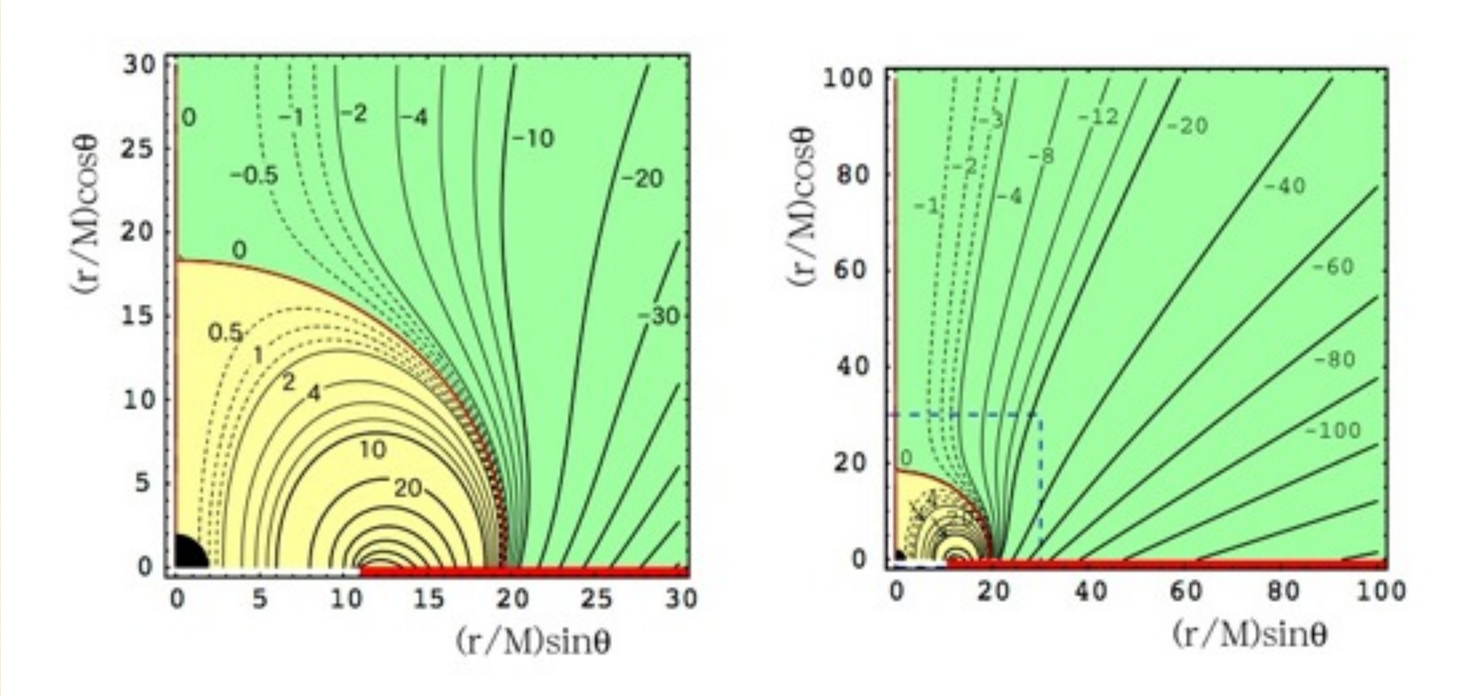


Angular Distribution of EM-Flux
We can see the Max. in equatorial region !

BZ-Power => Effect on Disk activity !

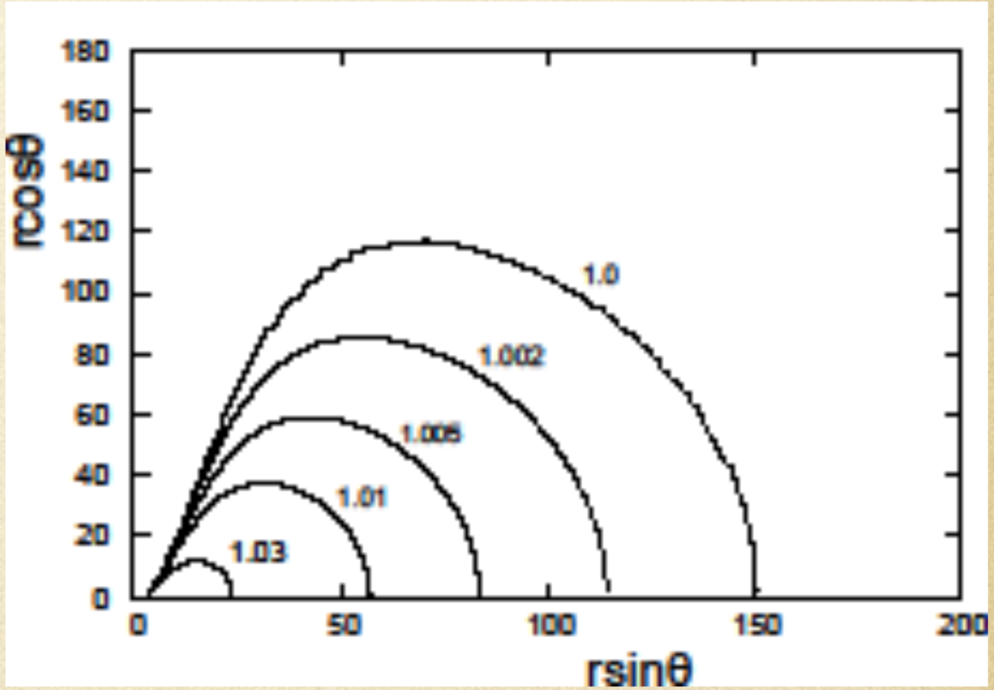
vacuum Maxwell sol.

Tomimatsu & MT 2000



一般相対論的流体分布

Fishbone 1977



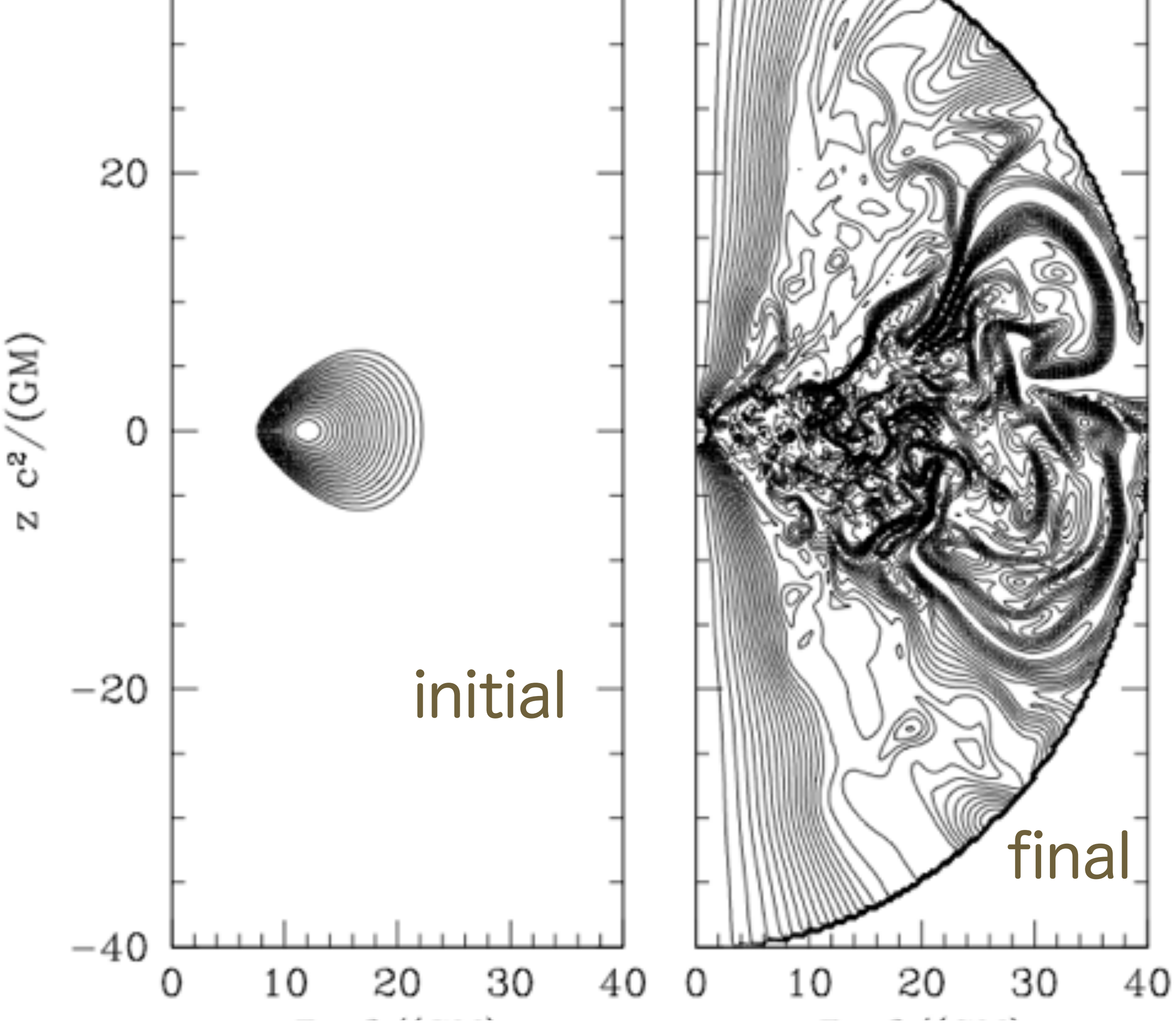
GRMHD Simulation

McKinney & Gammie 2004

Maeda et al. とは異なる結果

磁場強度分布と角速度分布

初期磁場依存性！



No. 2, 2004 KERR BLACK HOLE LUMINOSITY 989

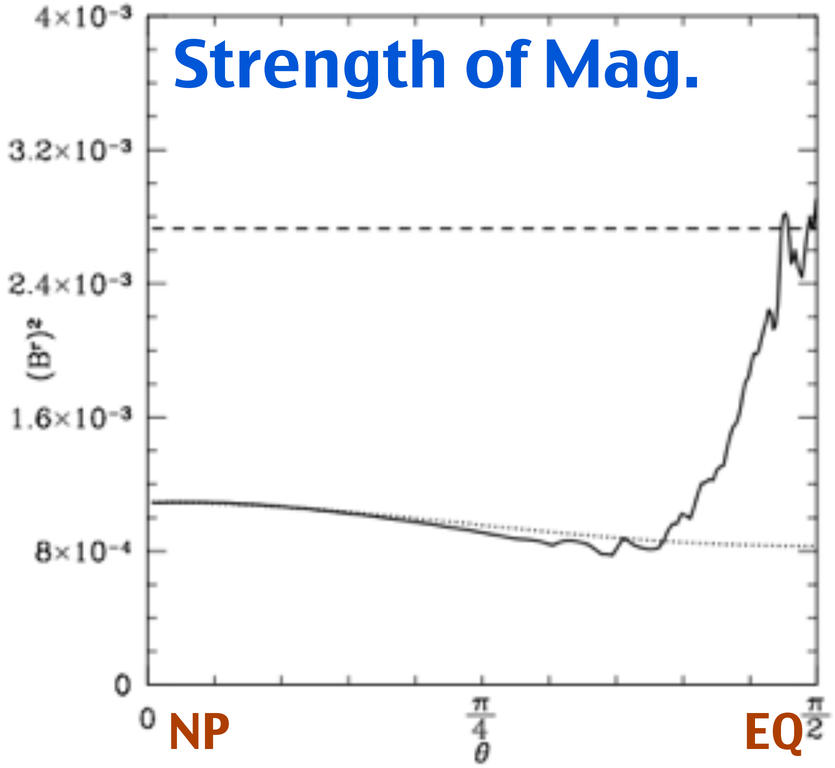


Fig. 9a

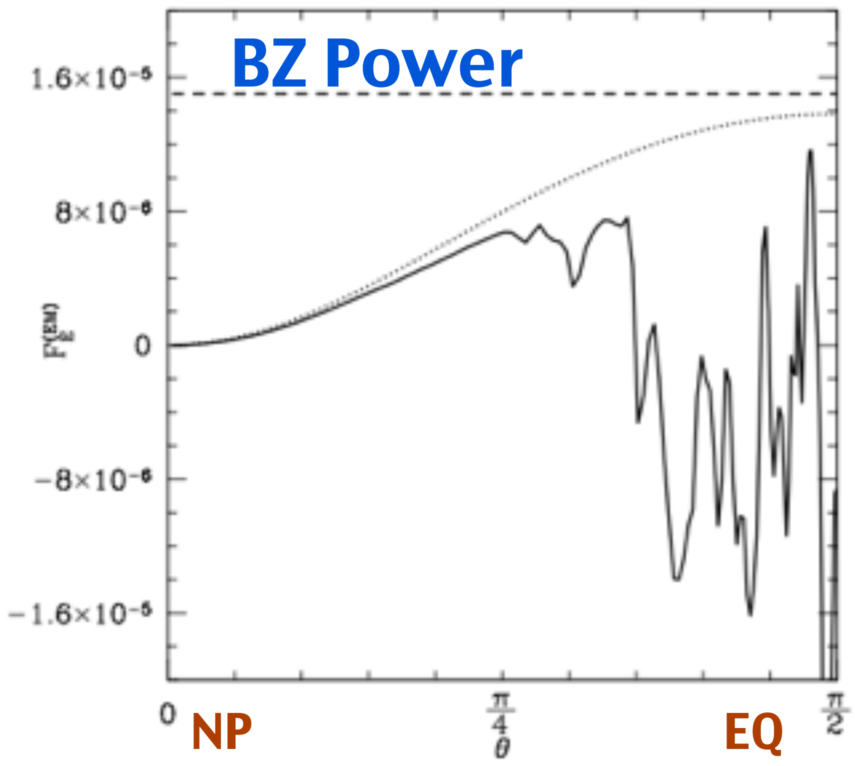
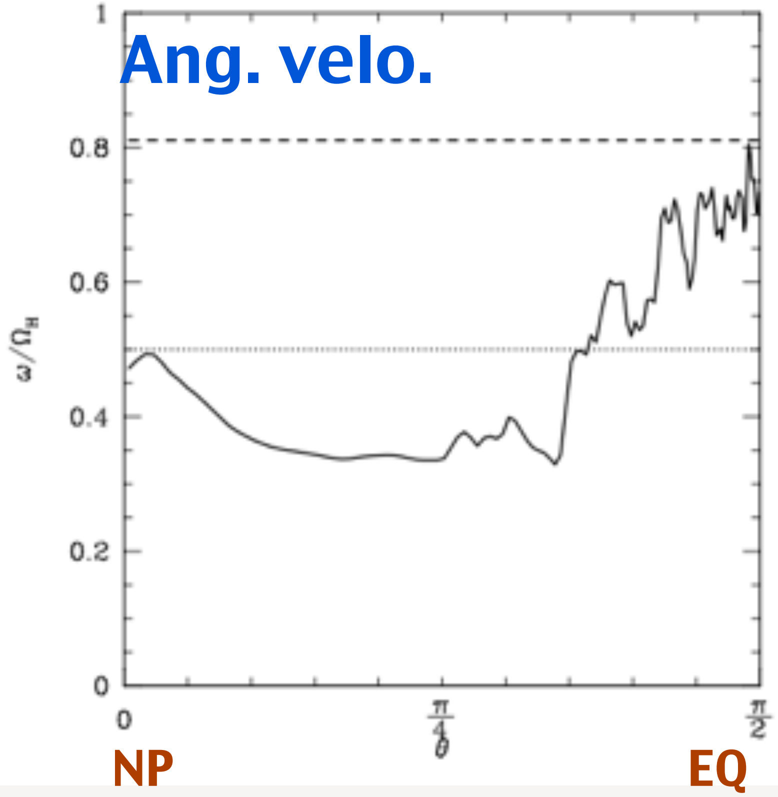
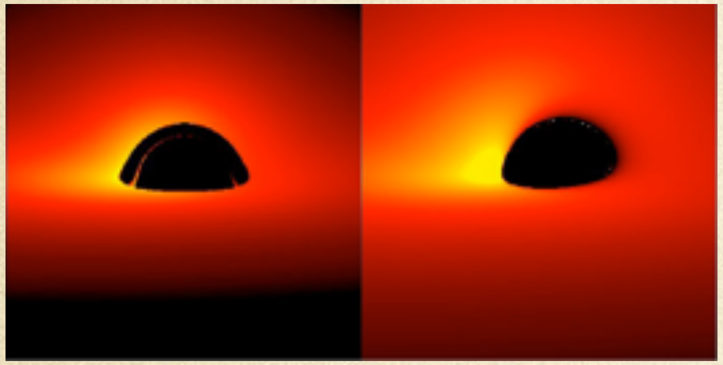


Fig. 9b



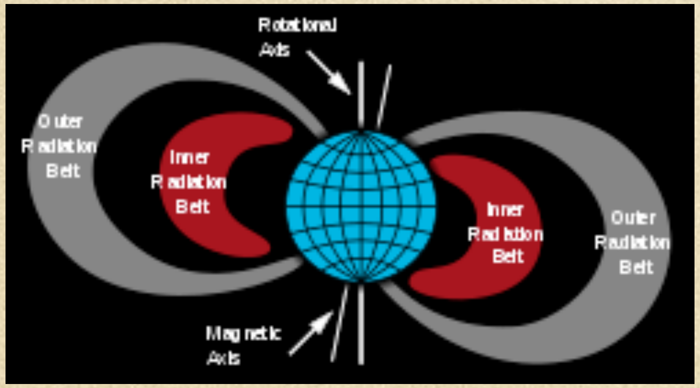
Information of the black hole spacetime

Accretion Disk --- **Image** of the Black Hole Shadow and Accretion Plasma (sub-mm VLBI)



BH - Aurora --- **HE-emission** from very close to the Event Horizon (X-ray, γ-ray)

BH - Van Allen radiation belt --- The plasma can be trapped in this zone, which may be related to a **cosmic ray** .



MHD wave propagation

in BH magnetosphere

Eikonal 近似

波の伝播を求める手順

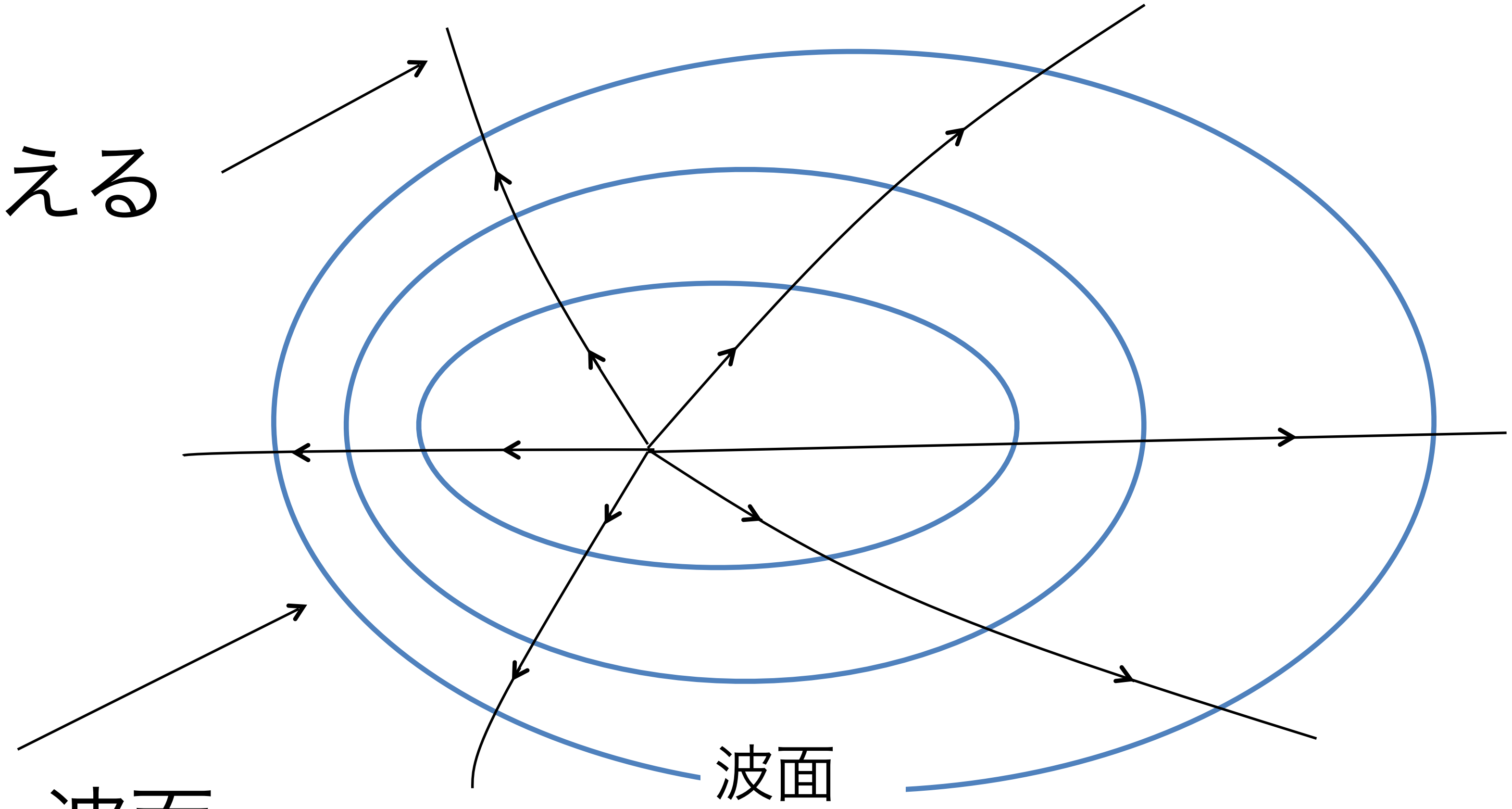
幾何光学近似
（波長が系の物理量の特徴的長さよりも十分に小さいとみなす近似）

フェルマーの原理より

波動方程式 → Eikonal 方程式 → 正準形式に書き直す

✿Eikonal方程式

波の特性曲線を与える
（射線）



時間の一定面＝波面

Eikonal方程式と有効ポテンシャル

形式的な表式

$$A^{\alpha\beta}p_\alpha p_\beta = 0$$

$A^{\alpha\beta}$ ： メトリック　p_α ：波の運動量

→

正準形式

$$\dot{x}^\alpha(\lambda) = \frac{\partial H(x^\alpha, p_\alpha)}{\partial p_\alpha}$$

$$\dot{p}_\alpha(\lambda) = -\frac{\partial H(x^\alpha, p_\alpha)}{\partial x^\alpha}$$

○光波の伝播（Kerr BHメトリック）

$$A^{\alpha\beta} = g^{\alpha\beta}$$

○音波の伝播（sound sonic メトリック）

$$A^{\alpha\beta} = s^{\alpha\beta} = g^{\alpha\beta} - \left(1 - \frac{1}{a_s^2}\right)u^\alpha u^\beta$$

a_s ：音波の速さ

$$a_s^2 = (\gamma - 1)\frac{h-1}{h}$$

u_α ： 流体の４元速度

●磁気流体波の伝播（Magnetosonic メトリック）

V_{MW} ： 磁気流体波の速さ

$$A^{\alpha\beta} = M^{\alpha\beta} = g^{\alpha\beta} - \left(1 - \frac{1}{V_{\mathrm{MW}}^2}\right)u^\alpha u^\beta$$

有効ポテンシャルを定義

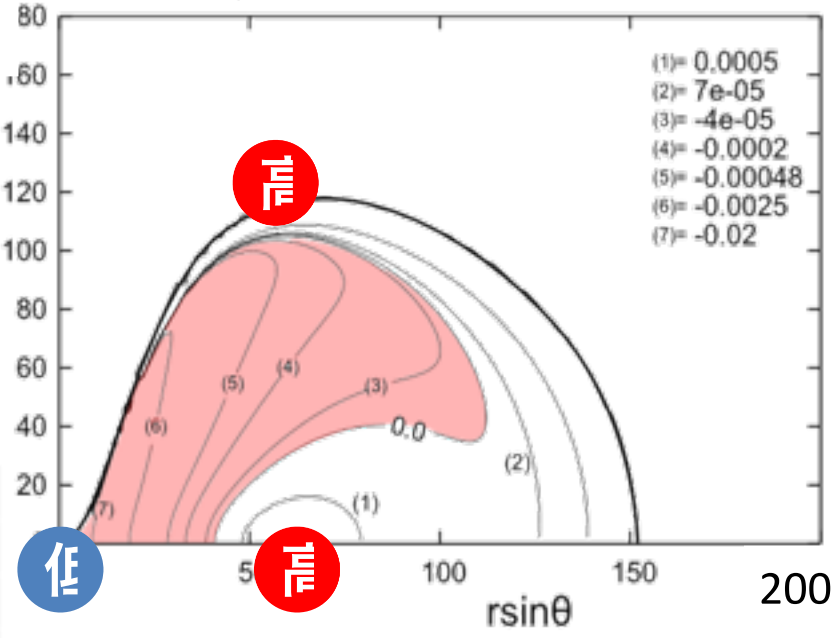
$$V_\pm^{1/2} \equiv \frac{E}{|L|} = \frac{\pm A^{t\phi} + \sqrt{(A^{t\phi})^2 - A^{tt}A^{\phi\phi}}}{A^{tt}}$$

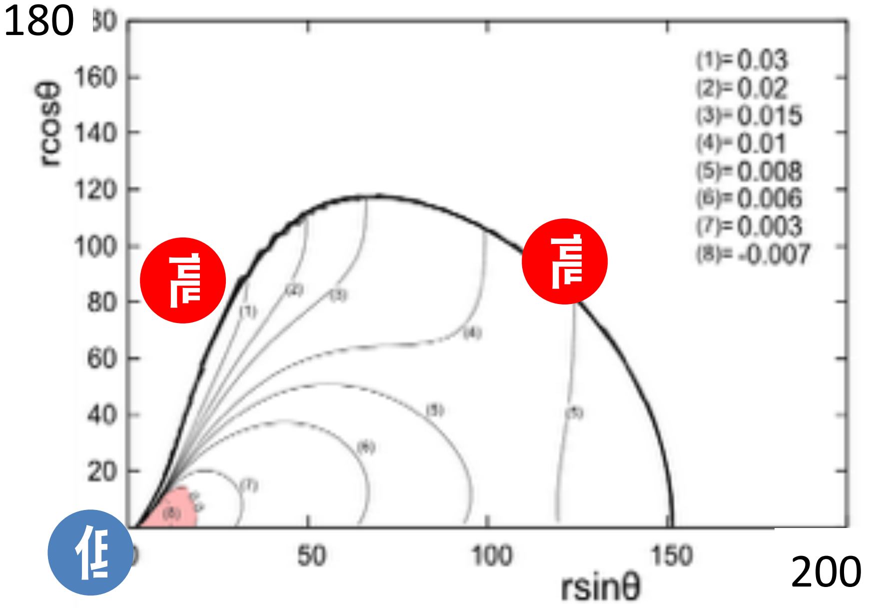
波の大まかな
伝播方向を議論

有効ポテンシャル（L<0）

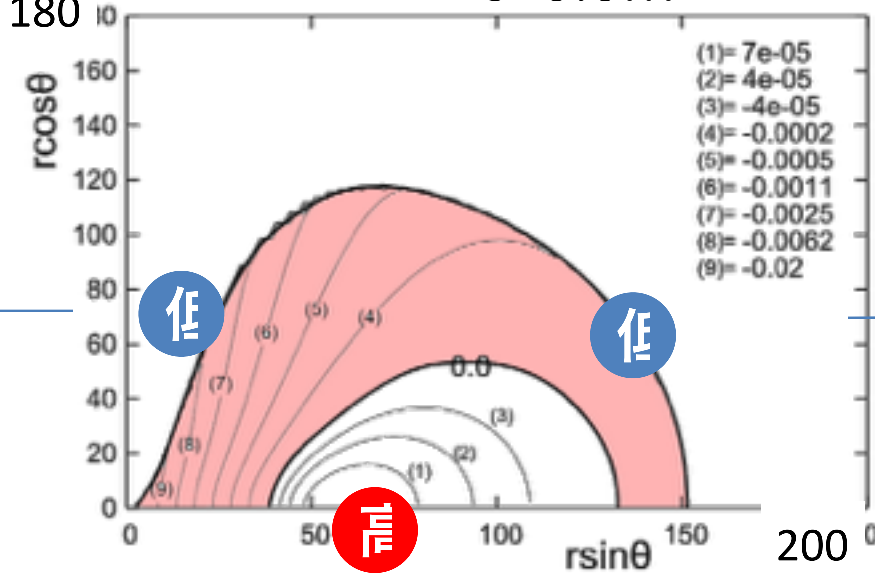
流体のパラメータ :$a = 0.8M, \underline{\ell = 3.0M,} r_{\rm in} = 2.85M, \gamma = 4/3$

Paraboloidal磁場強度分布

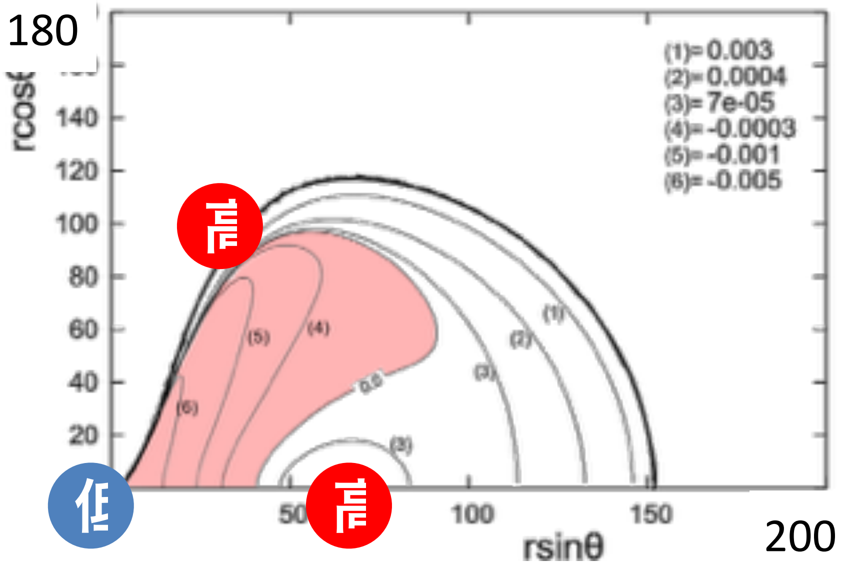

C=1.0M

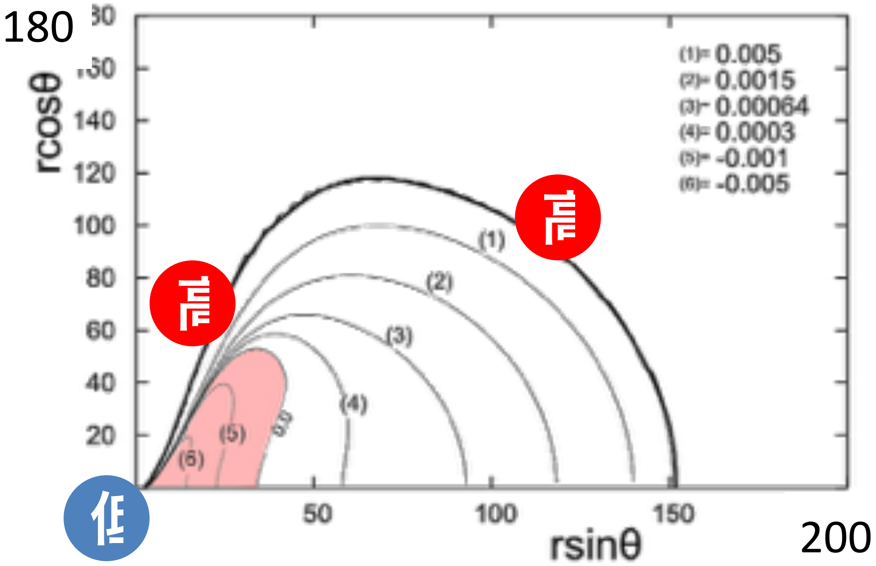

C=6.0M

音波


C=0.0M

磁場強度を強くする


C=3.0M


C=20M

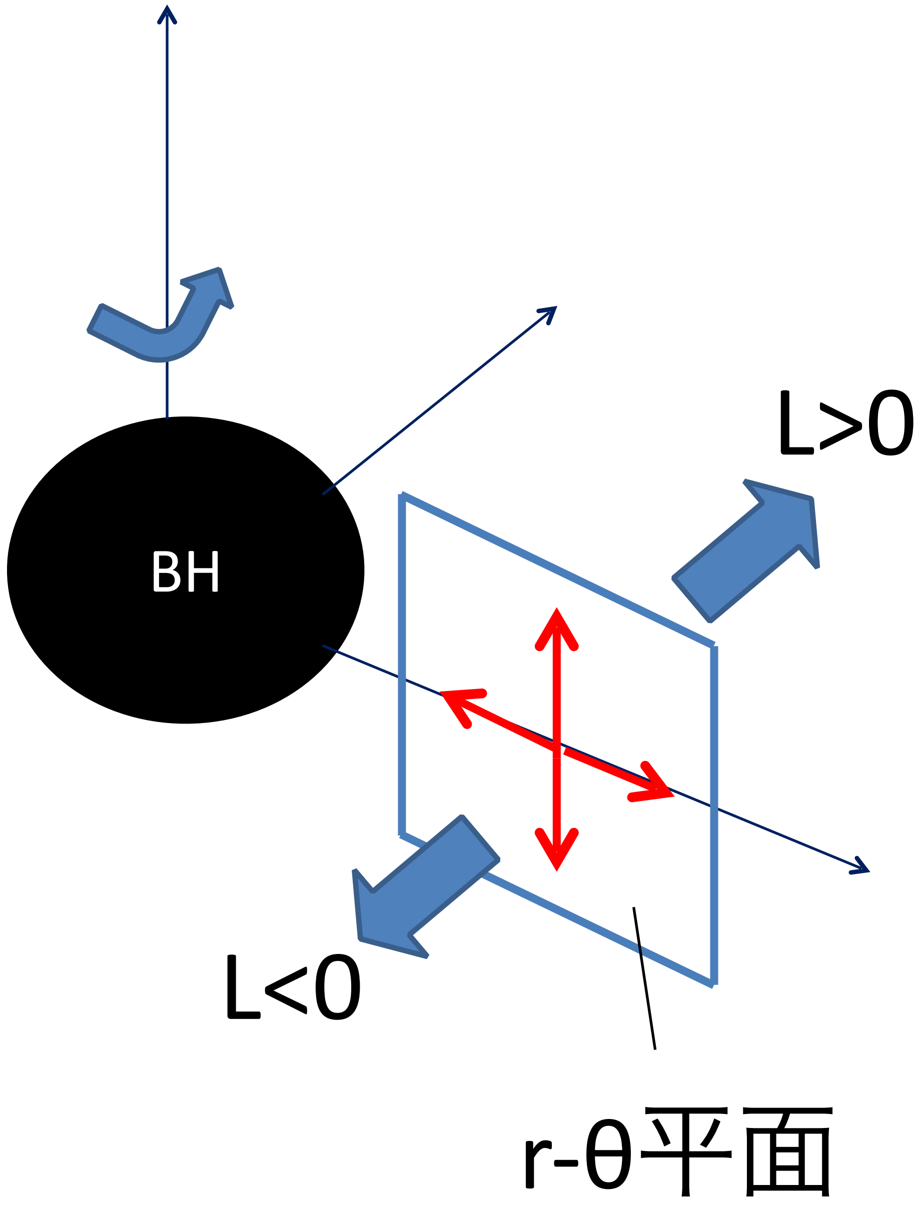
Split-monopole磁場強度分布

有効ポテンシャルと波の伝播

L>0　BHと同じ方向の運動量成分（正の角運動量）

L<0　BHと逆方向の運動量成分（負の角運動量）

波の初期値の与え方



エネルギー、角運動量は一定

$$L = -p_\phi = \text{const}$$

$$E = p_t = \text{const}$$

p_θの決め方

$$(p_\theta)_\pm \equiv \pm\sqrt{-(g^{tt}p_tp_t + 2g^{t\phi}p_tp_\phi + g^{\phi\phi}p_\phi p_\phi)/g^{\theta\theta}}$$

$$(p_\theta)_- \leq p_\theta \leq (p_\theta)_+$$

p_rの決め方

$$p_r = \pm\sqrt{-(g^{tt}p_tp_t + 2g^{t\phi}p_tp_\phi + g^{\theta\theta}p_\theta p_\theta + g^{\phi\phi}p_\phi p_\phi)/g^{rr}}$$

磁気流体波の伝播（L<0）

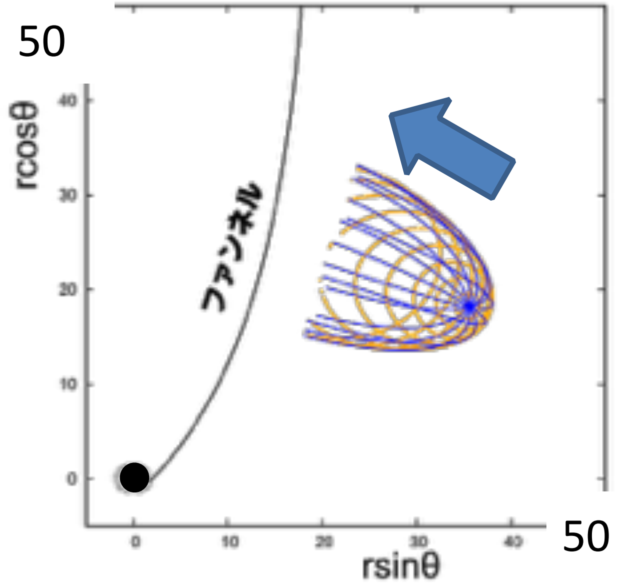
青：射線　橙：波面

流体のパラメータ：$a = 0.8M, \underline{\ell = 3.0M,} r_{\rm in} = 2.85M, \gamma = 4/3$

Paraboloidal磁場強度分布

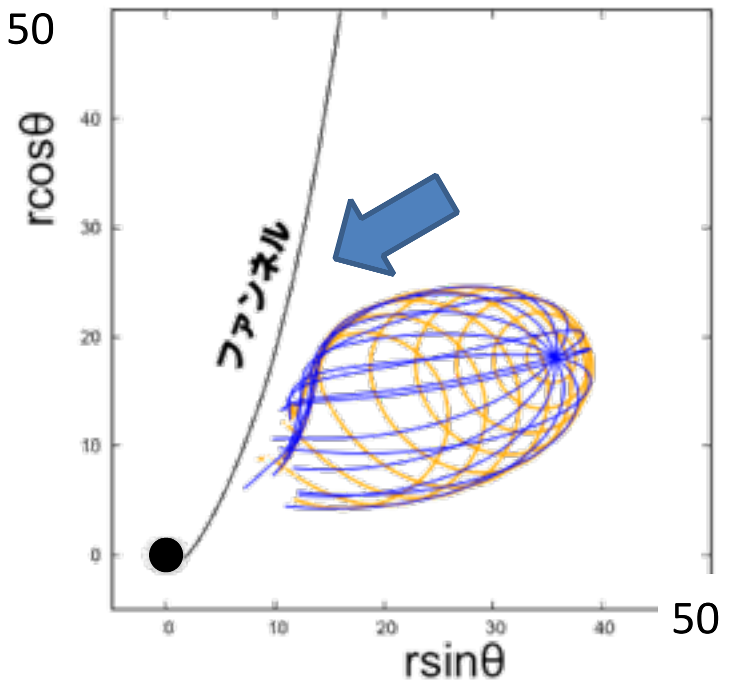
音波

C=0.0M

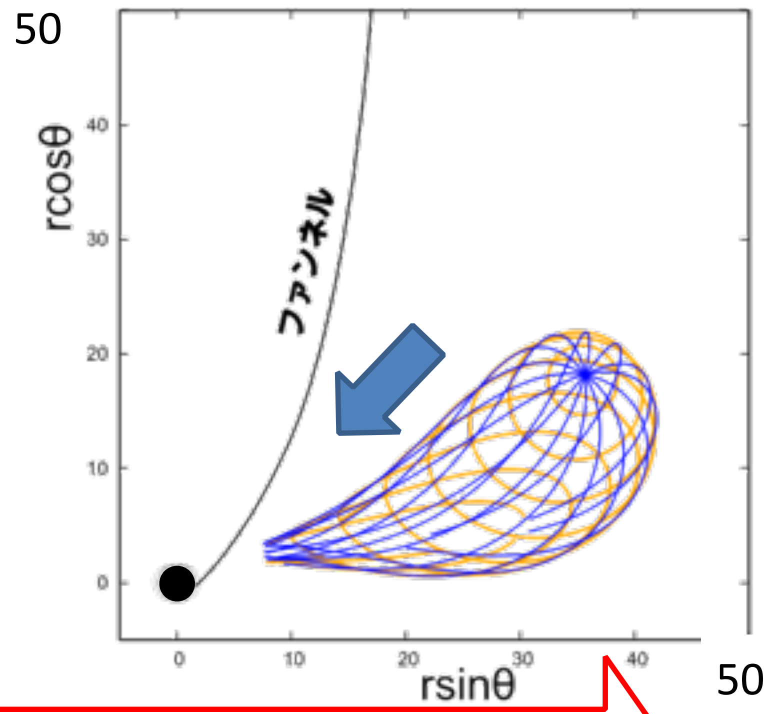


Split-monopole磁場強度分布

C=2.0M

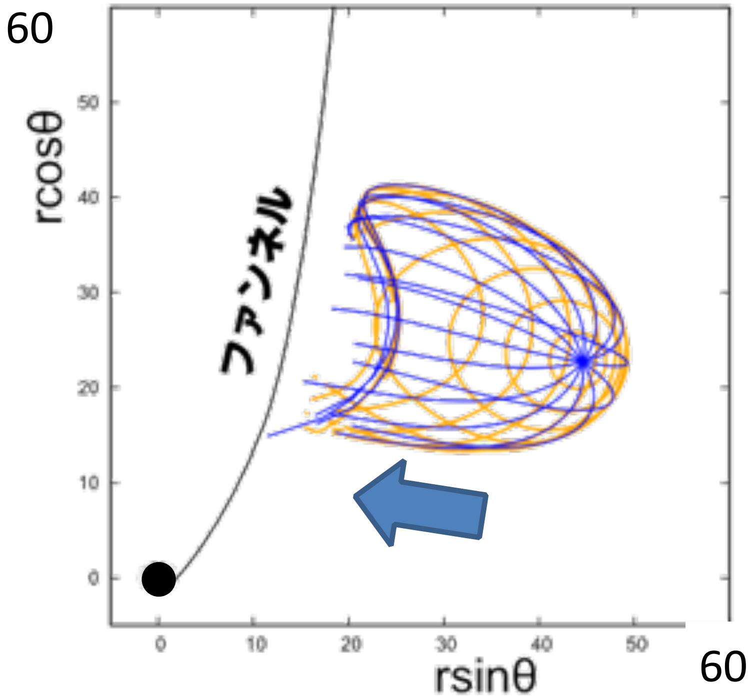


C=6.0M

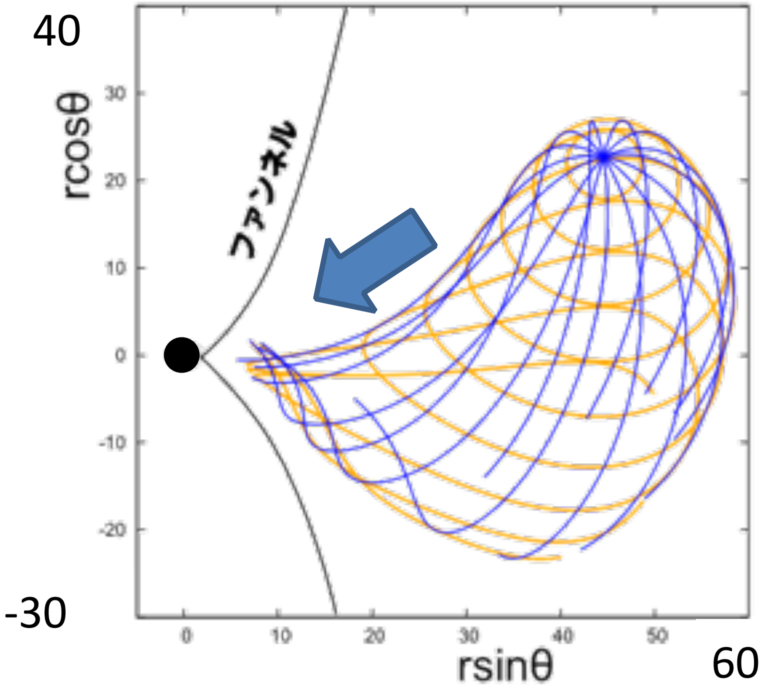


磁場強度を強める

C=20M



C=60M

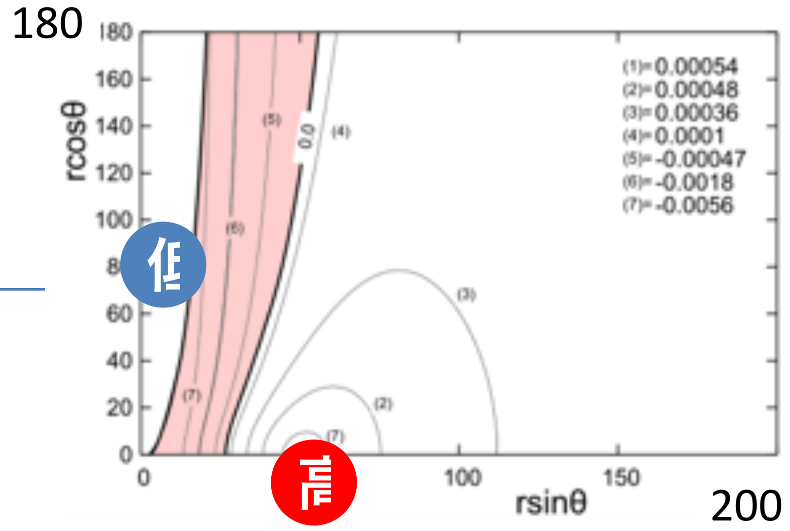


有効ポテンシャル（L<0）

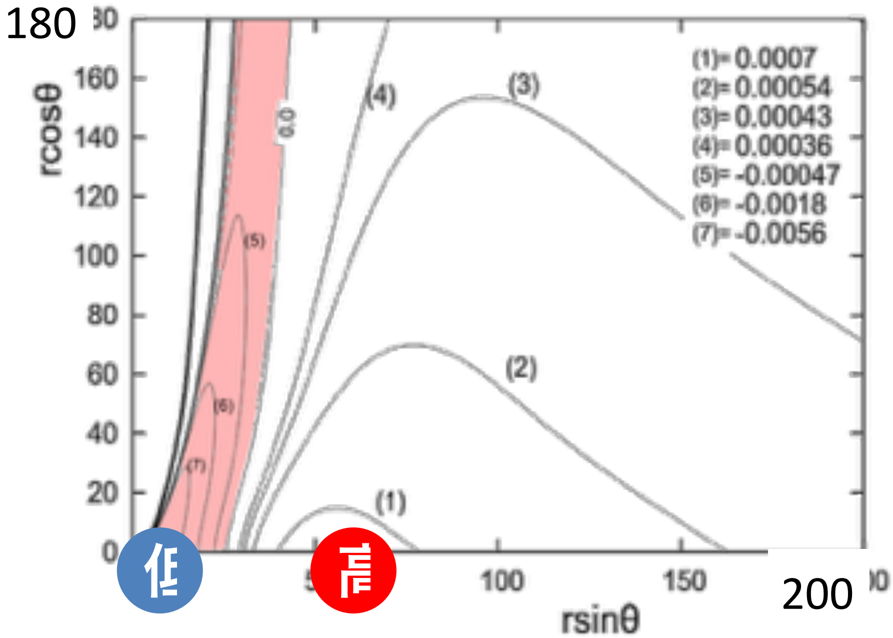
流体のパラメータ： $a = 0.8M, \underline{\ell = 3.06M}, r_{\text{in}} = 2.85M, \gamma = 4/3$

Paraboloidal磁場強度分布

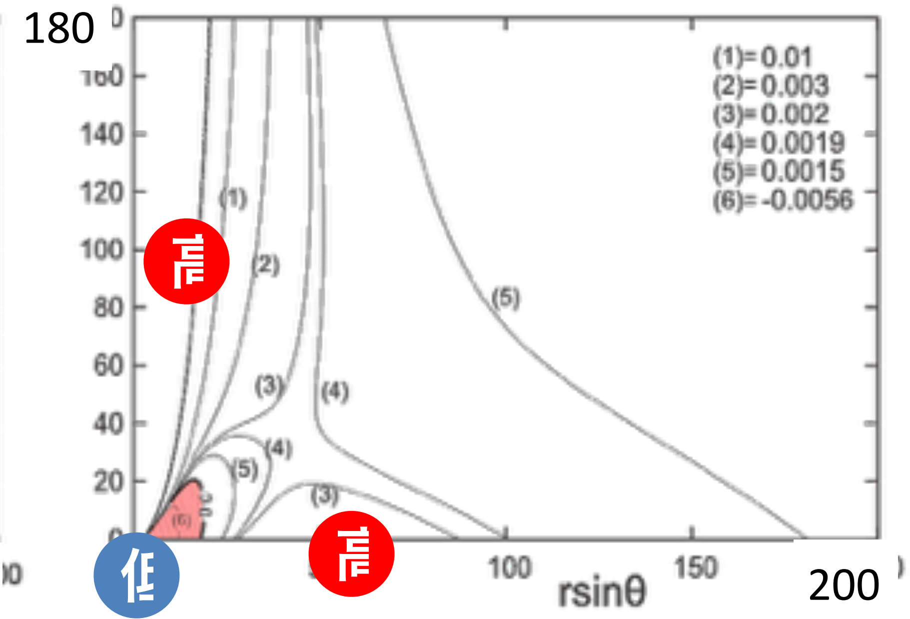
Split-monopole磁場強度分布



C=2.0M

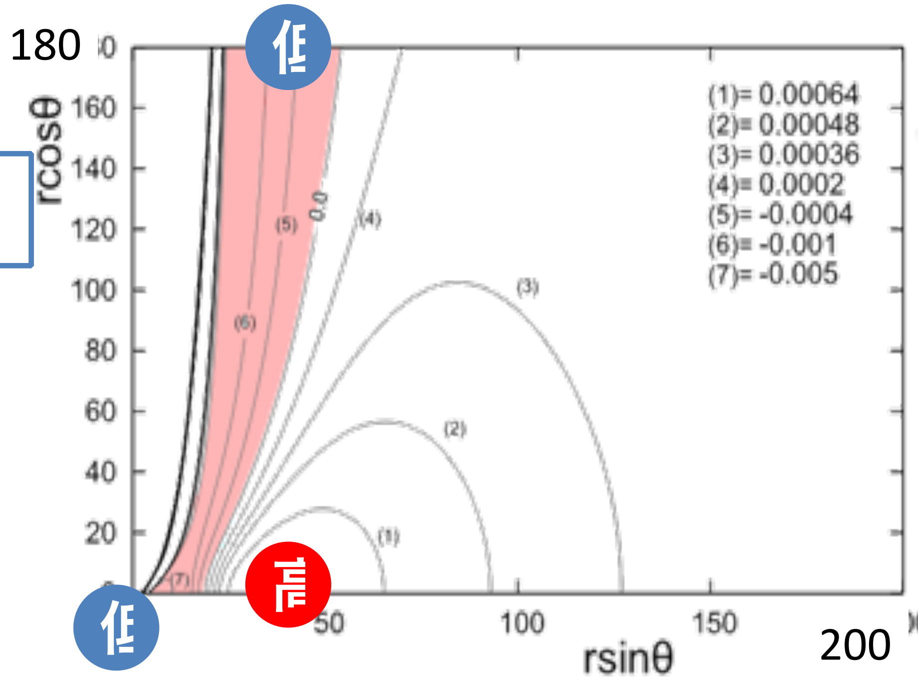


C=6.0M

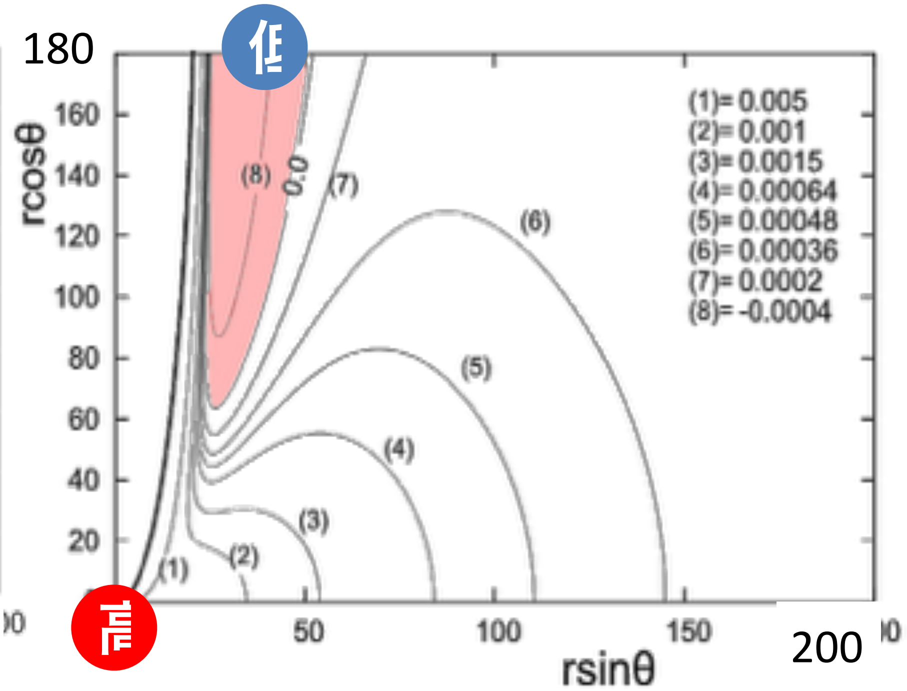


磁場強度を強める

C=70m



C=110m



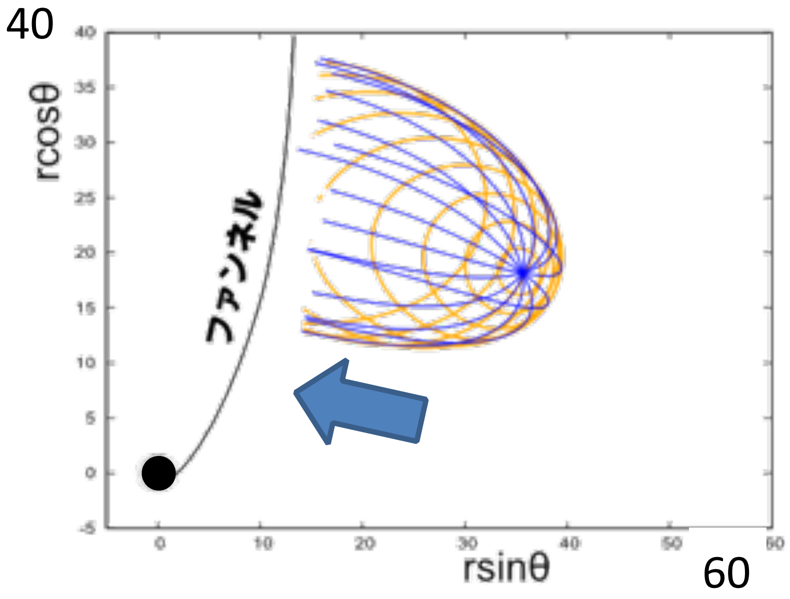
磁気流体波の伝播（L<0）

青：射線　橙：波面

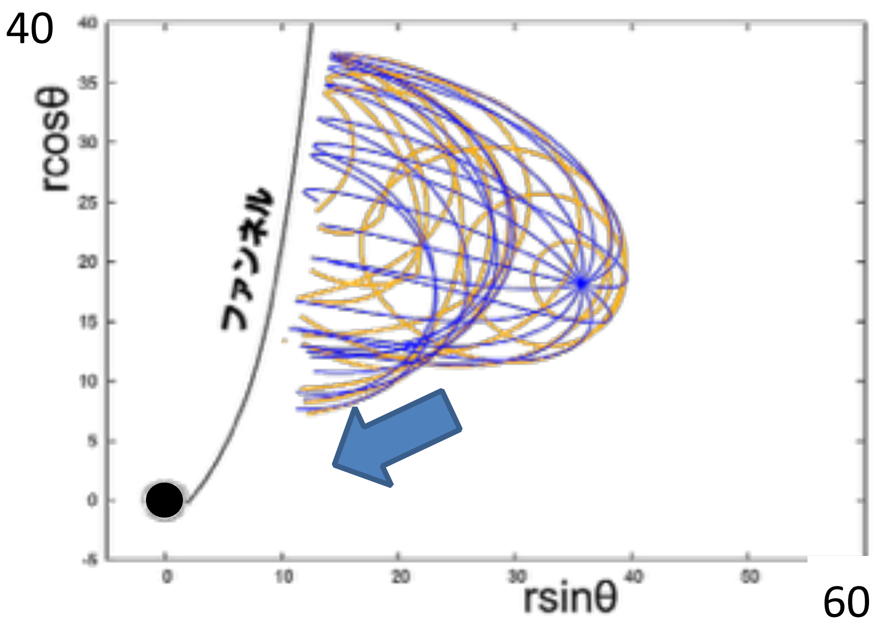
流体のパラメータ：　$a = 0.8M, \underline{\ell = 3.06M}, r_{\rm in} = 2.85M, \gamma = 4/3$

Paraboloidal磁場強度分布

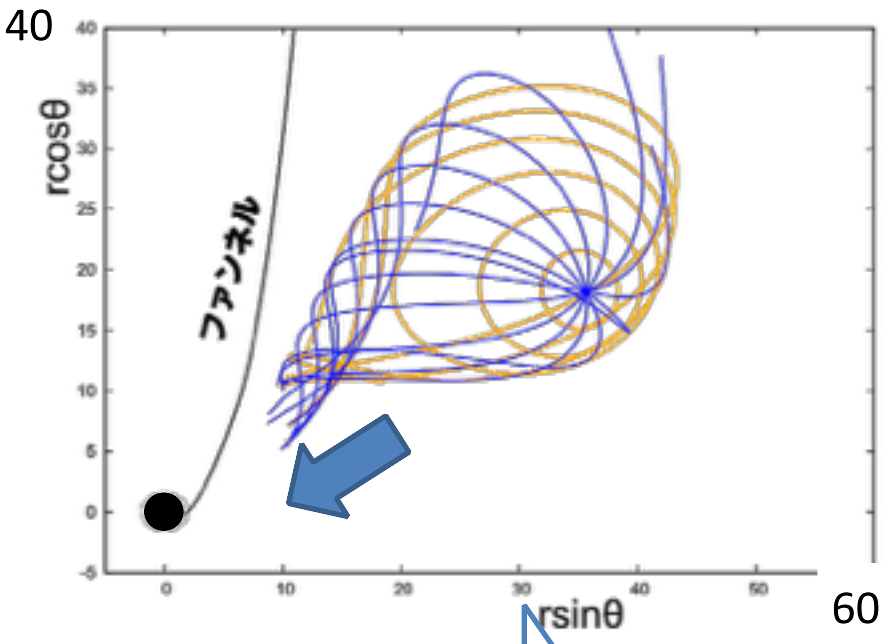
C=0



C=2.0M



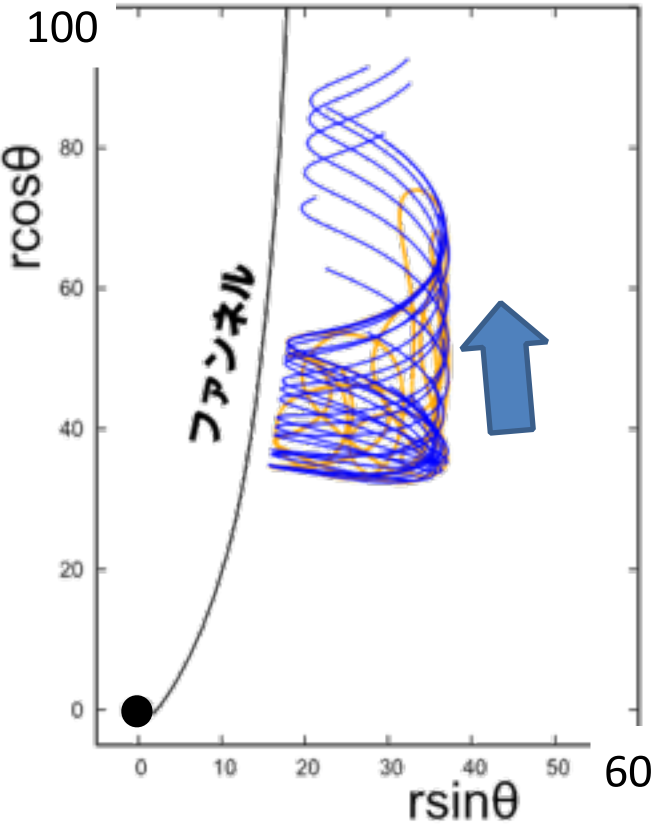
C=6.0M



磁場強度を強める

Split-monopole磁場強度分布

C=70M



C=110M