Brief Introduction to the Scheme utilized in CMFGEN

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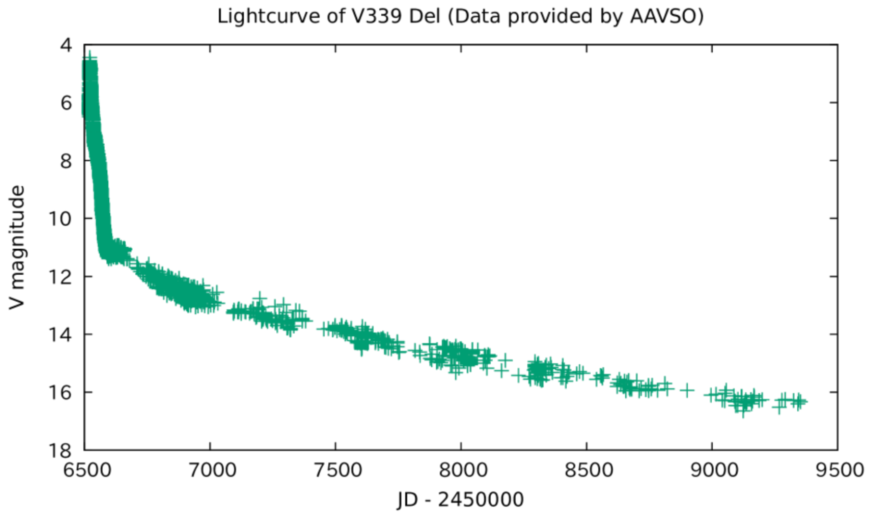
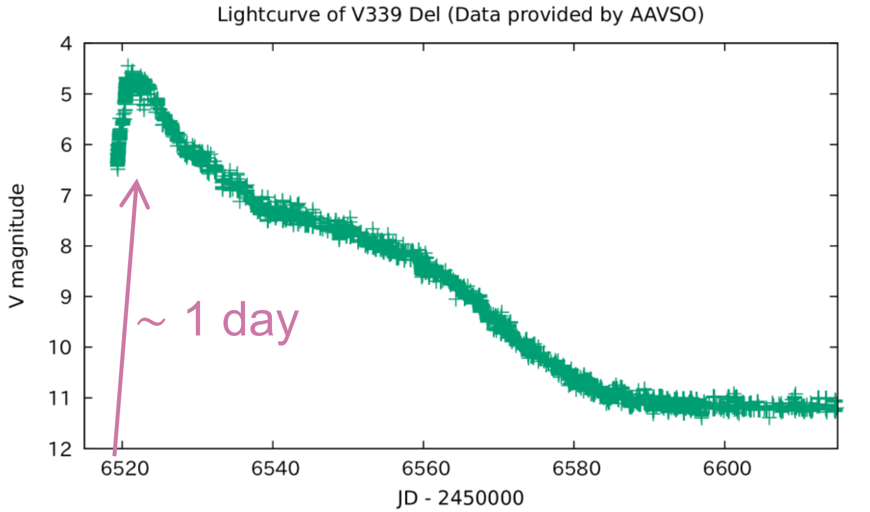
(Classical) Novae: (古典) 新星、新星爆発

- Historically, a nova means “a new star” in Latin.
  - Sudden (~ 1 d) appearance of apparently “new” star.
- However, novae are not caused by a new stars, but they are transient events on a existing stars.
  - They typically brighten by ~ 10 mag in V-band.
    - At least by ~ 7 mag, at maximum ~ 19 mag.
  - They gradually fade typically for weaks - years.
- Their maximum absolute V magnitudes are between −6 and −10.

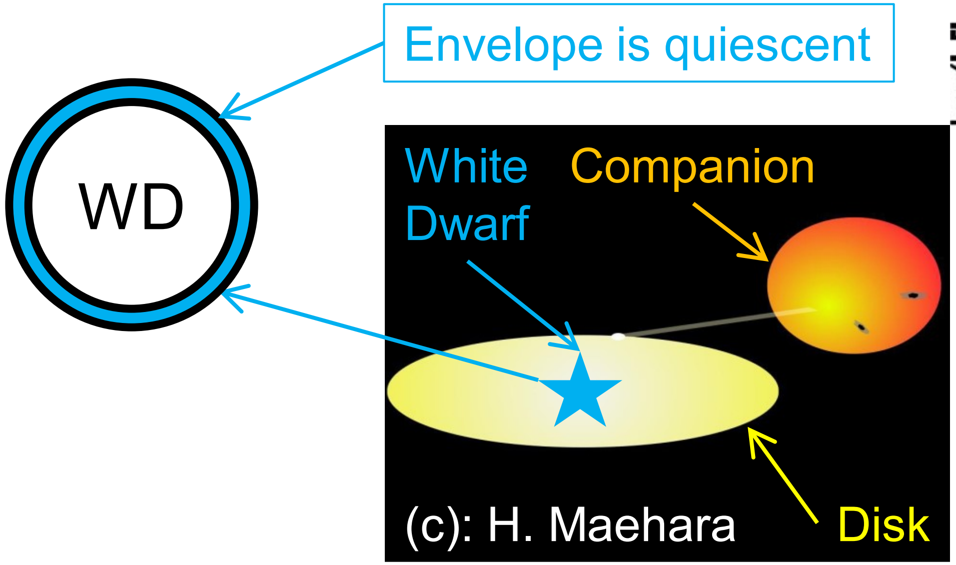
  → Observable novae: only in our galaxy, MCs (+ M31, etc).
  - Too faint compared to supernovae.
  - Too bright compared to dwarf novae.

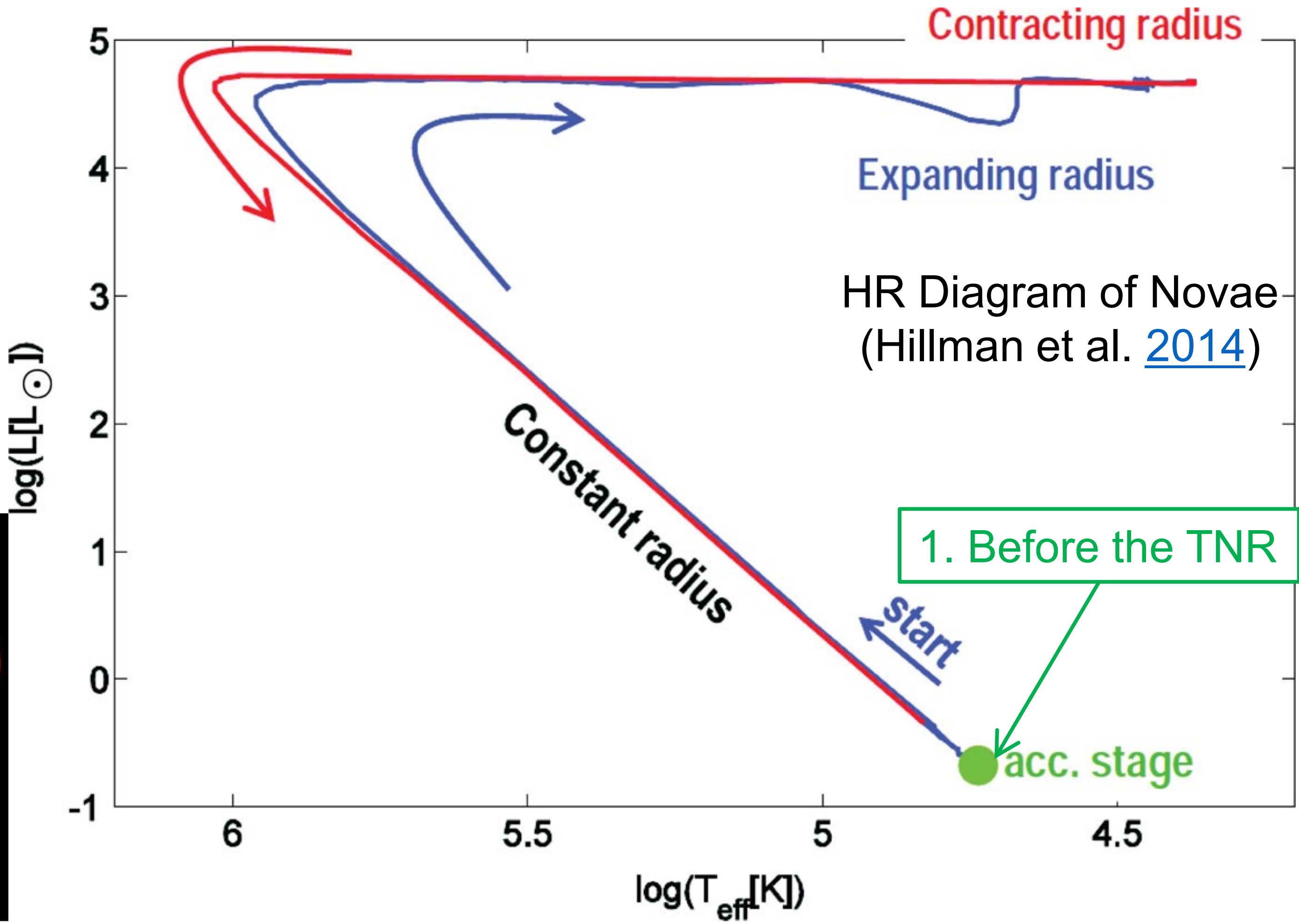




Nova Cycle in the HR Diagram (Hillman et al. 2014, Kato, Saio and Hachisu 2017a)

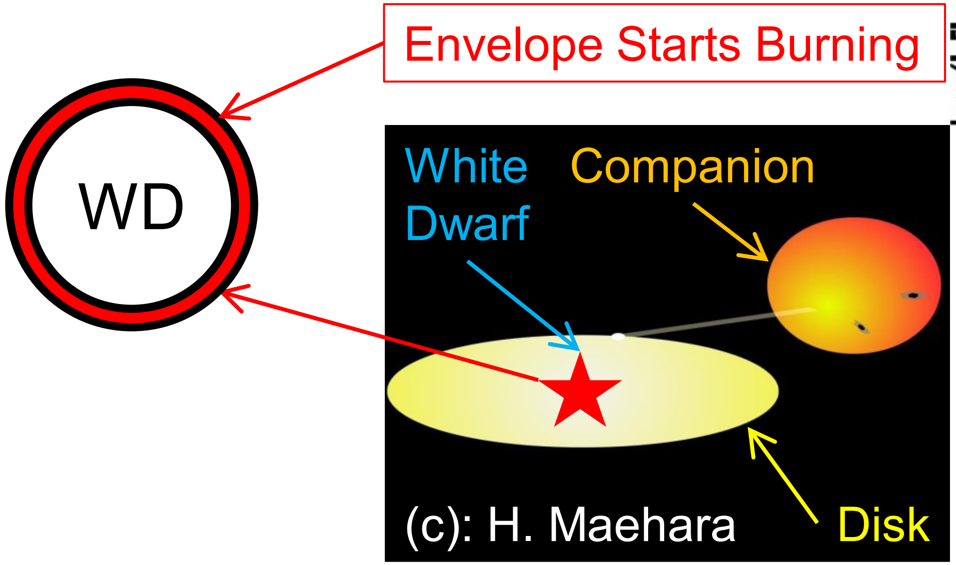
1. Before TNR, WD is faint.

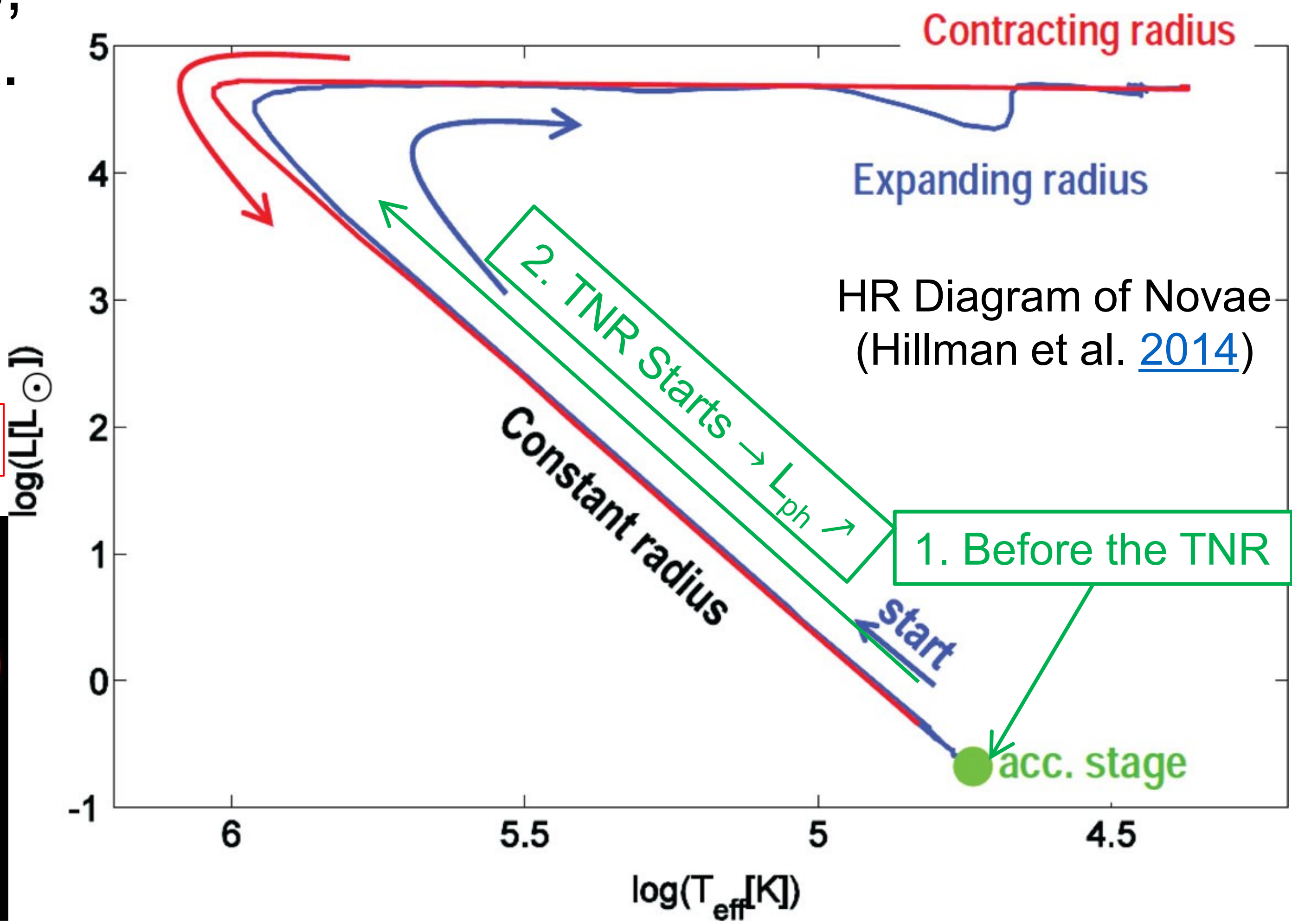




Nova Cycle in the HR Diagram (Hillman et al. 2014, Kato, Saio and Hachisu 2017a)

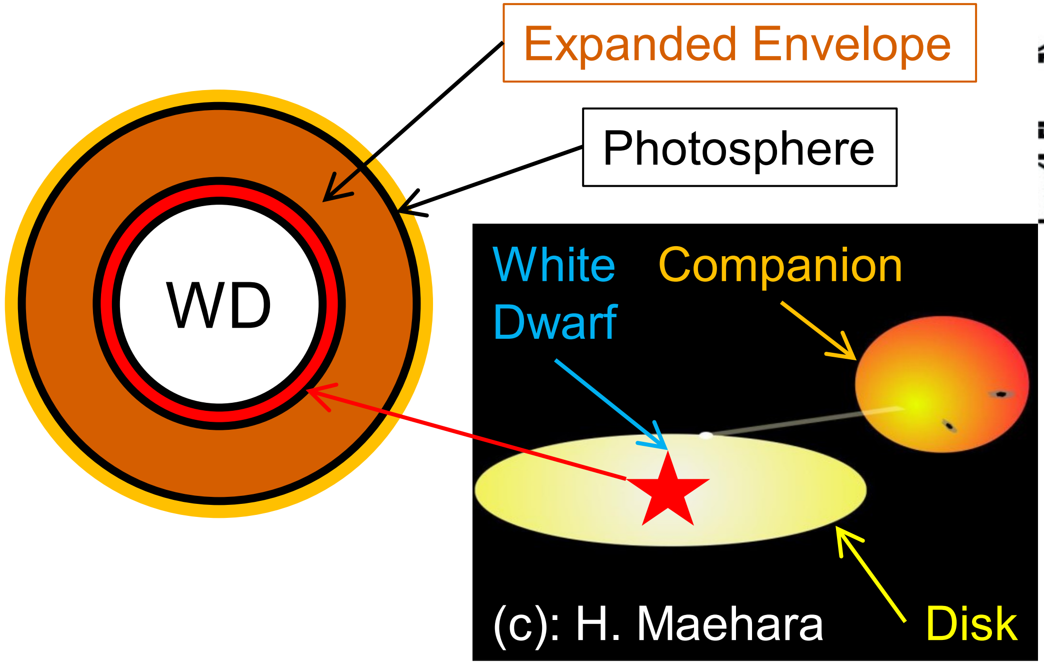
2. Once the nuclear burning starts, $L_{ph} \nearrow$, keeping R_{ph} almost const.

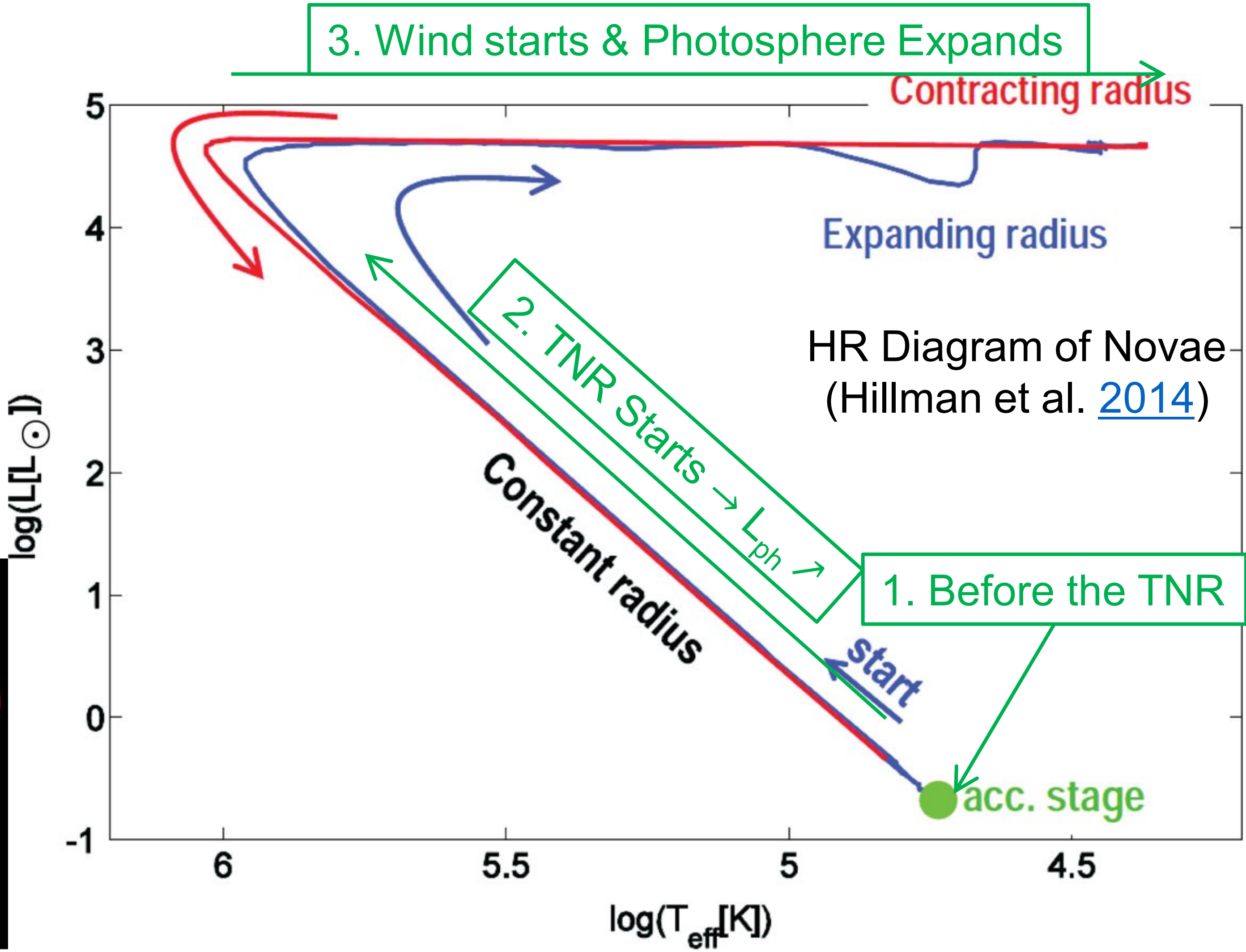




Nova Cycle in the HR Diagram
(Hillman et al. 2014, Kato, Saio and Hachisu 2017a)

3. After L_{ph} reaches almost L_{Edd}, optically thick wind occurs and photosphere expands.
   - $L_{ph} \approx L_{Edd}$ and $R_{ph} \nearrow$ (so $T_{eff} \searrow$)





Nova Cycle in the HR Diagram (Hillman et al. 2014, Kato, Saio and Hachisu 2017a)

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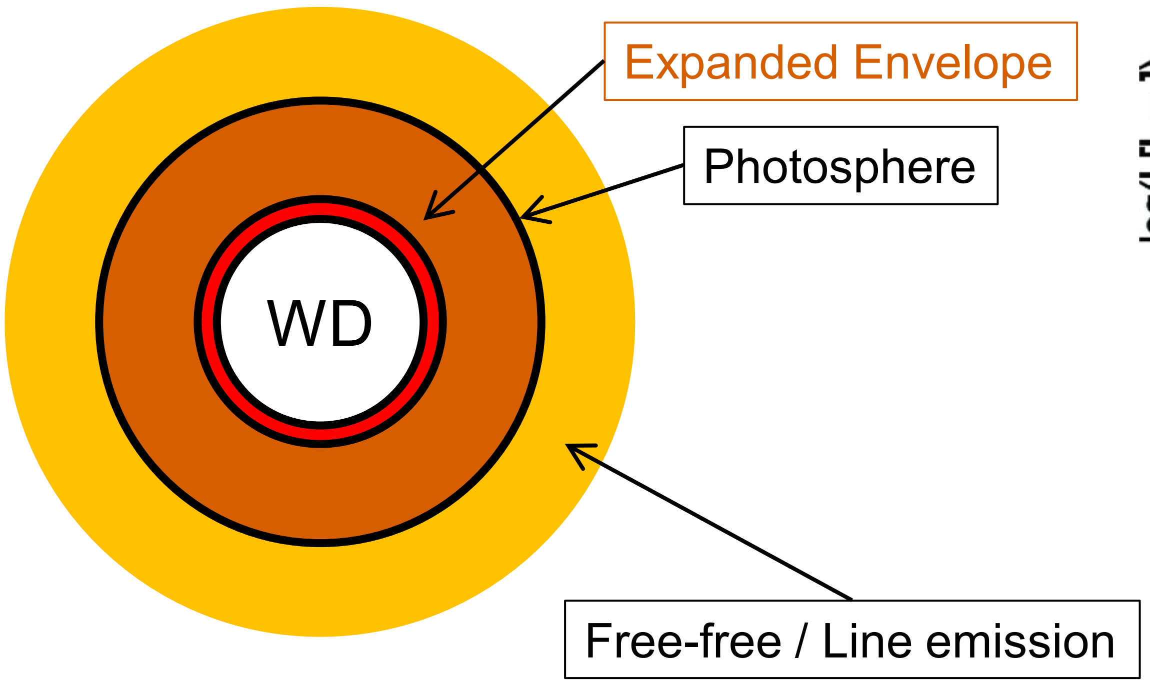
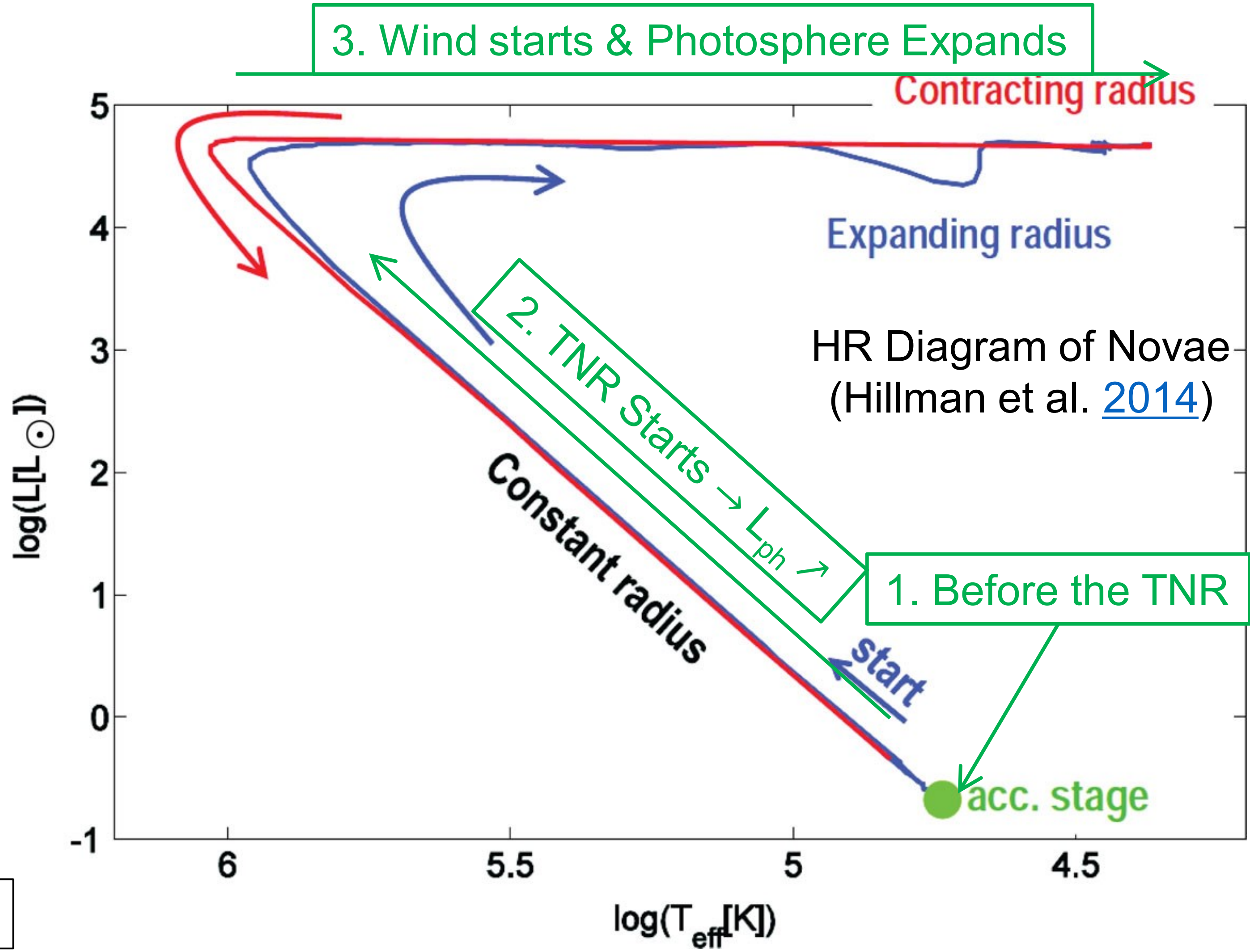
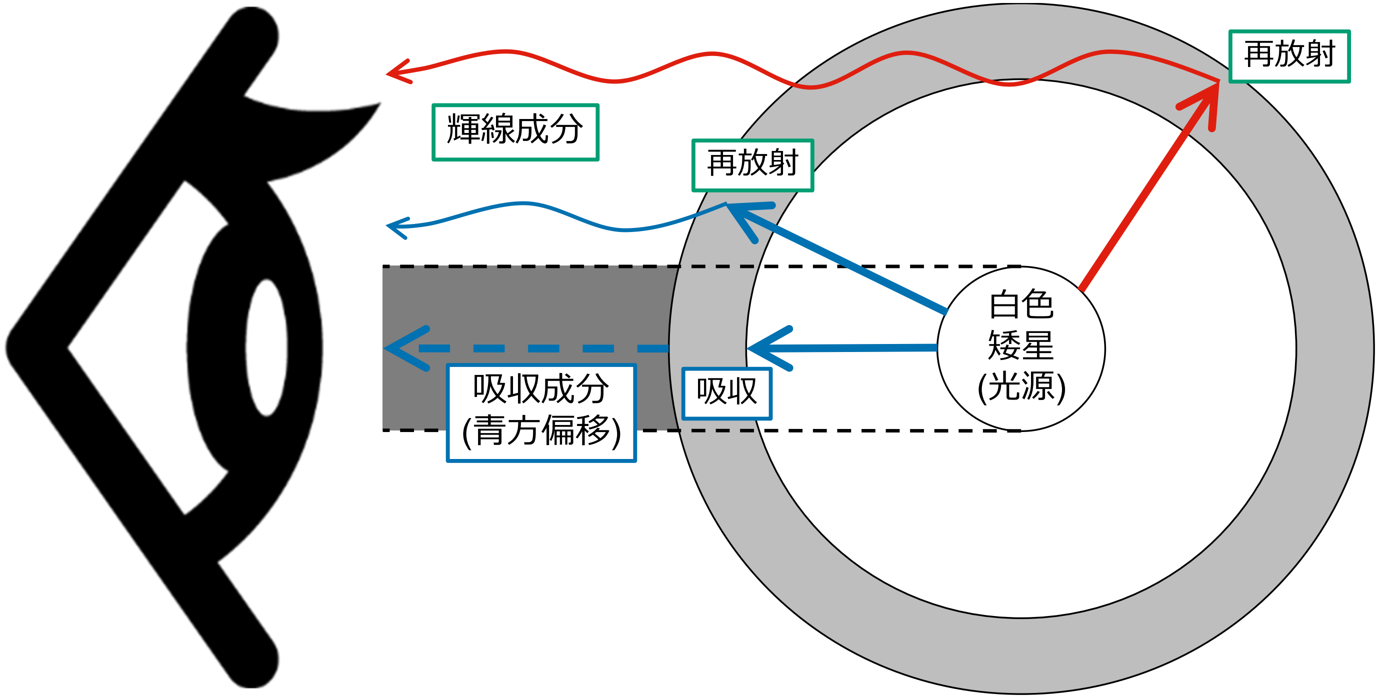
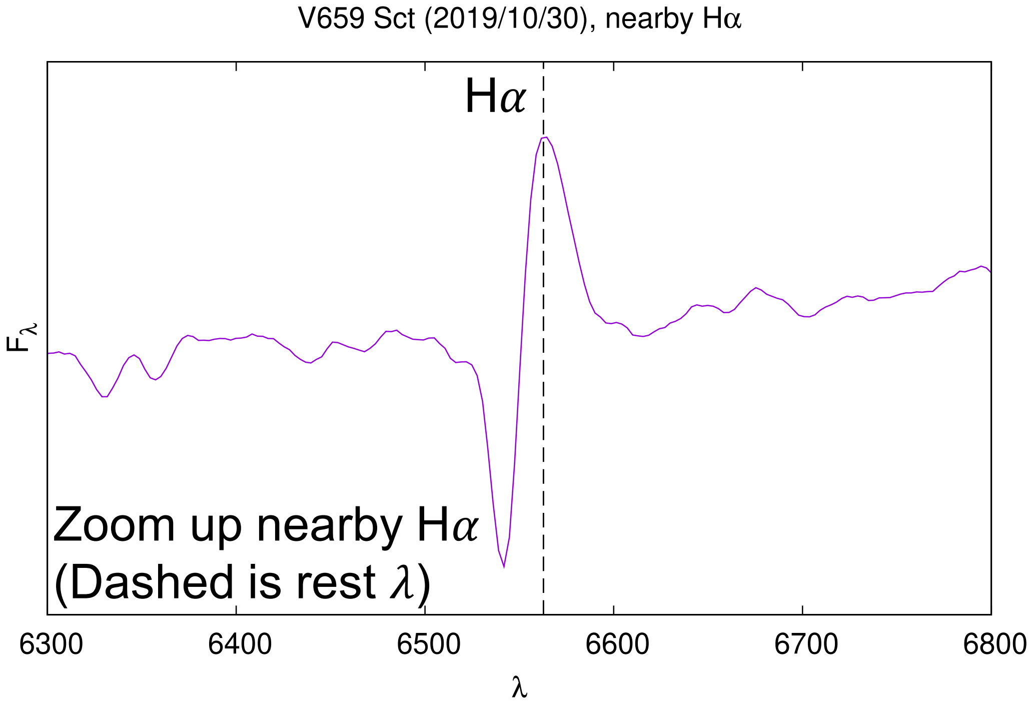




Spectrum of Nova V659 Sct, 1 day after its Discovery (Observed in Seimei Openuse 19B-N-CT03)



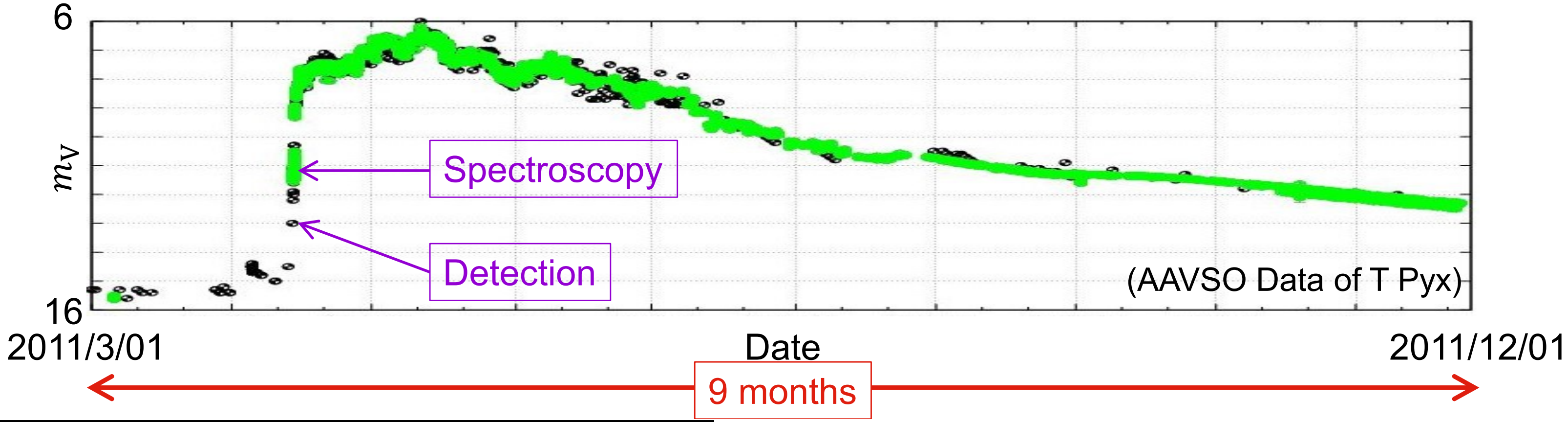
- Nearby optical peak magnitude, many lines of P Cygni profiles are in spectrum
  - Blue-shifted absorption
  - Emission at almost rest wavelength

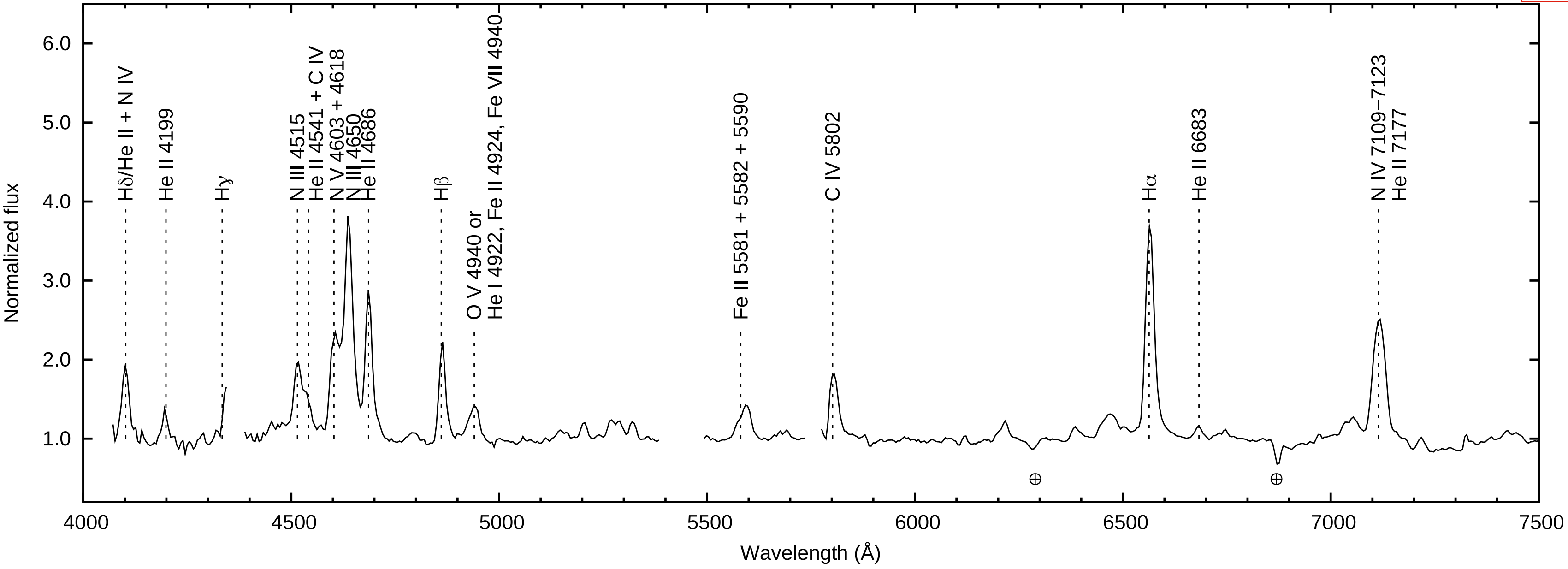


- This P Cygni profile suggests optically thick winds.
  → This nova had optically thick wind, 1 day after its discovery.
- This is consistent with the "optically thick wind theory" by Kato & Hachisu.

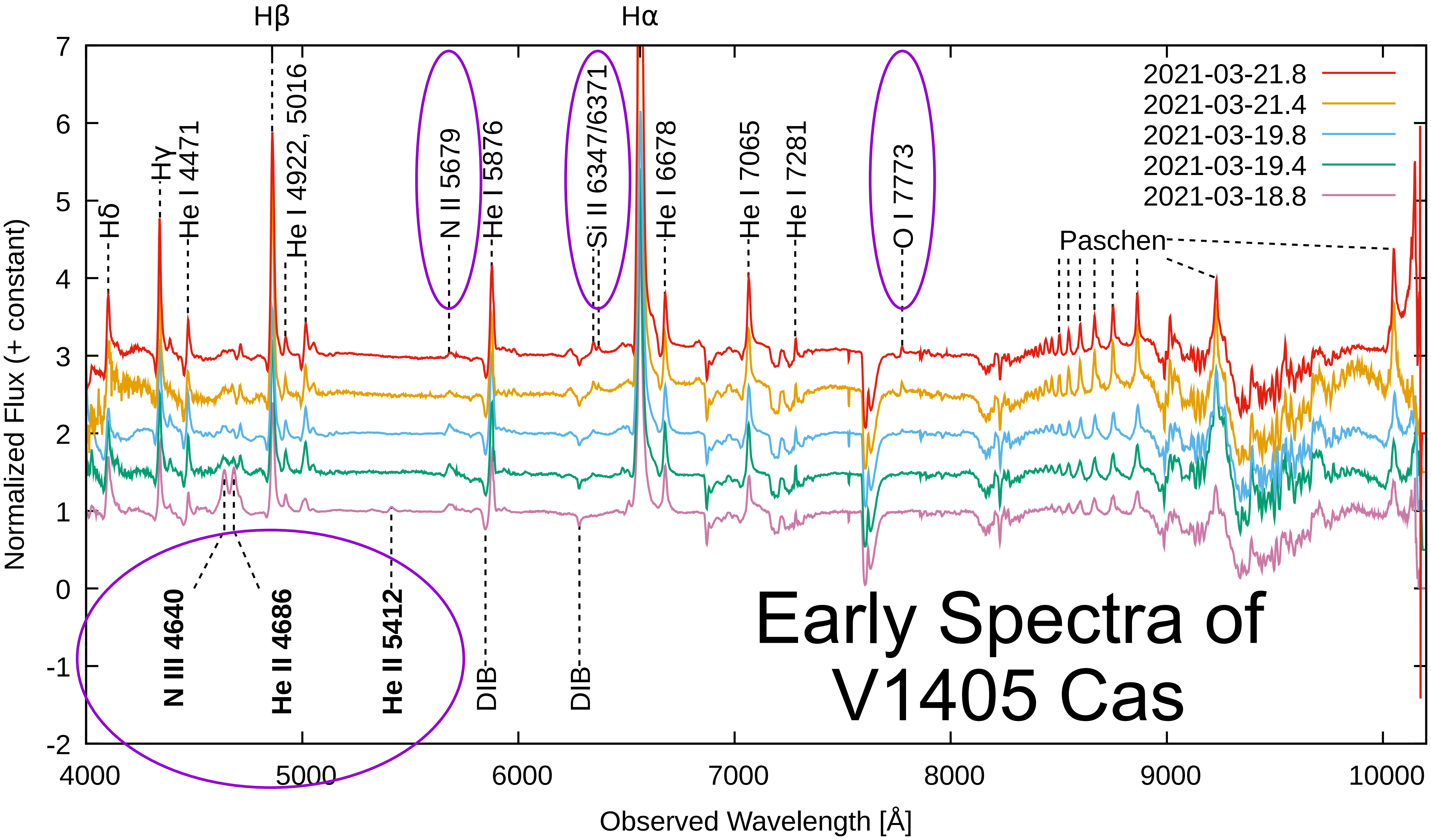
A Super-early Follow-up of the Recurrent Nova T Pyx (Arai et al. 2015)

- In 2011, a spectrum of T Pyx is taken only 4 hours after its discovery (around ~ 11 mag).
  - It is a known recurrent nova, so many people had monitored it.



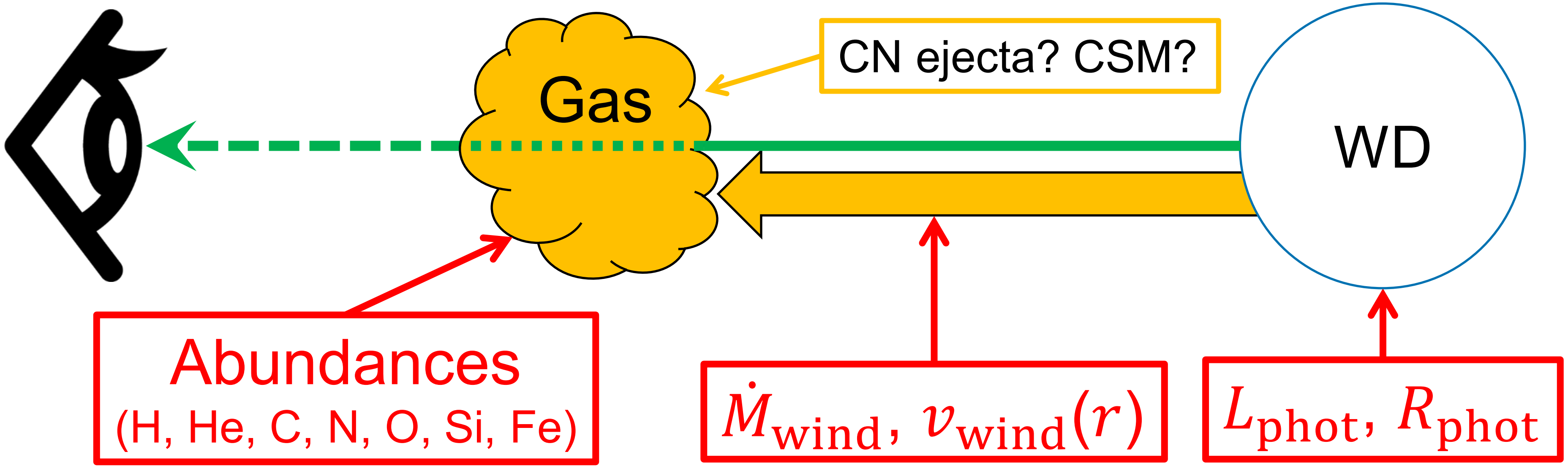


- Highly-ionized emission lines
  - Not P Cygni profile
- May contain the information of the system before optically thick wind fully evolved.


Hβ
Hα
2021-03-21.8
2021-03-21.4
2021-03-19.8
2021-03-19.4
2021-03-18.8
Normalized Flux (+ constant)
Observed Wavelength [Å]
Hδ
Hγ
He I 4471
He I 4922, 5016
N II 5679
He I 5876
Si II 6347/6371
He I 6678
He I 7065
He I 7281
O I 7773
Paschen
N III 4640
He II 4686
He II 5412
DIB
DIB
Early Spectra of
V1405 Cas

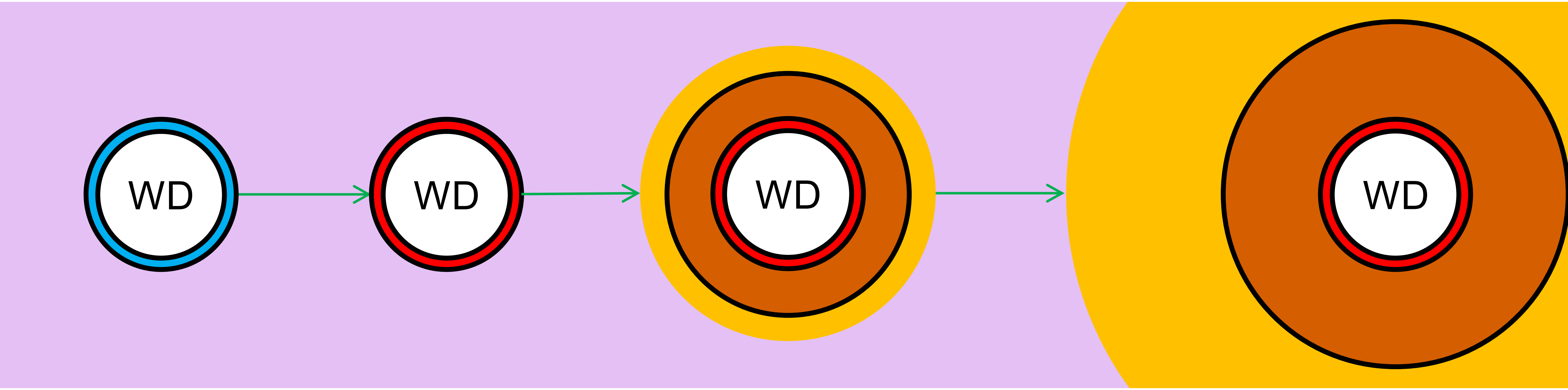
About CMFGEN

- We use the radiative transfer code CMFGEN (Hillier and Miller 1998) to calculate the expected spectra of novae of model systems.
  - This code solves non-LTE rate equation, radiative transfer equation, and electron temperature self-consistently in spherical geometry.
  - We regard the structure of nova system (White Dwarf + maybe ejecta) as the same of that of wind-blowing stars, which this code prefers.
  - We approximate nova system steady (like H II regions).



Nova is an Expanding System

- Nova photosphere expands typically upto ~ 100 $R_\odot$ (~ 1000 km/s × 1 day)
  - Ejecta front continues to expand (to ~ $10^3/10^4$ $R_\odot$ in ~ 10/100 days)



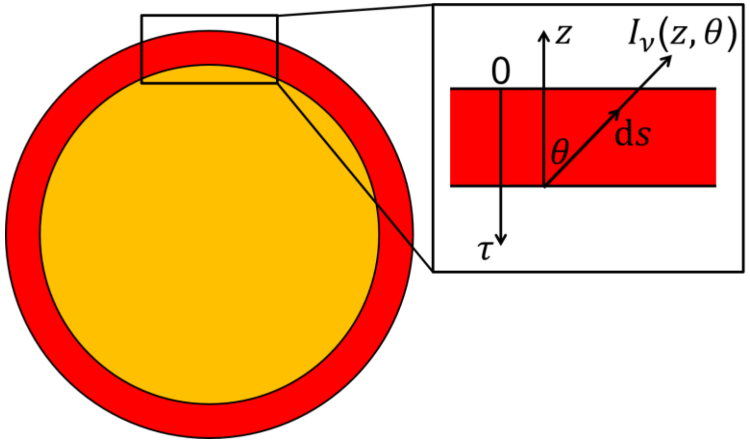
Three Requirements in Computing Nova Spectra

- Requirements to the transfer equation ($\frac{\mathrm{d}I_\nu}{\mathrm{d}s} = -\alpha_\nu I_\nu + j_\nu$)
  - Spherical Geometry
  - Blowing Winds

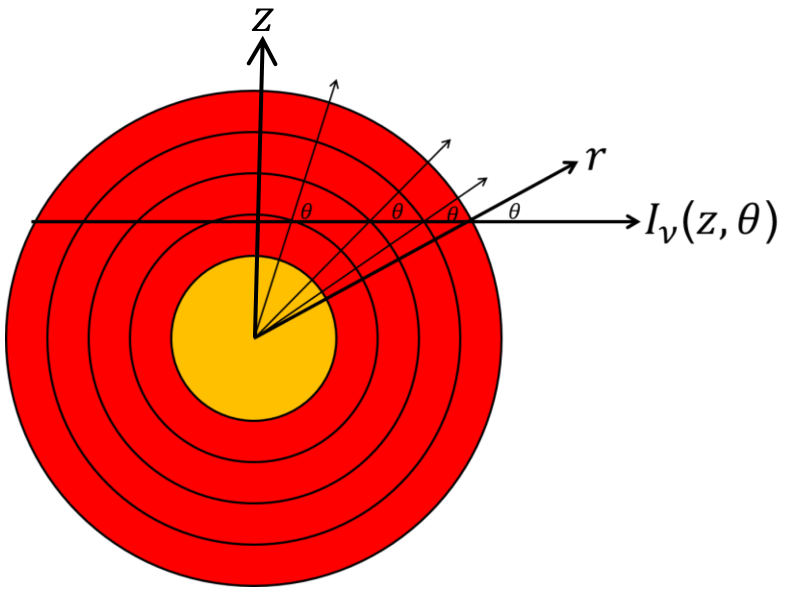
- Requirements to the absorbability/emissivity:
  - Non-LTE

Plane Parallelism vs Spherical Geometry

- Plane Parallelism
  - The upper plane is effectively parallel to the bottom plane.
  - So, $\mathrm{d}z = \mathrm{d}s\cos\theta$ or $\frac{\mathrm{d}}{\mathrm{d}s} = \cos\theta\frac{\mathrm{d}}{\mathrm{d}z}$
    - Solar Photosphere (~ 10^{11} cm) ≫ Chromosphere (~ $10^{8\text{-}9}$ cm)

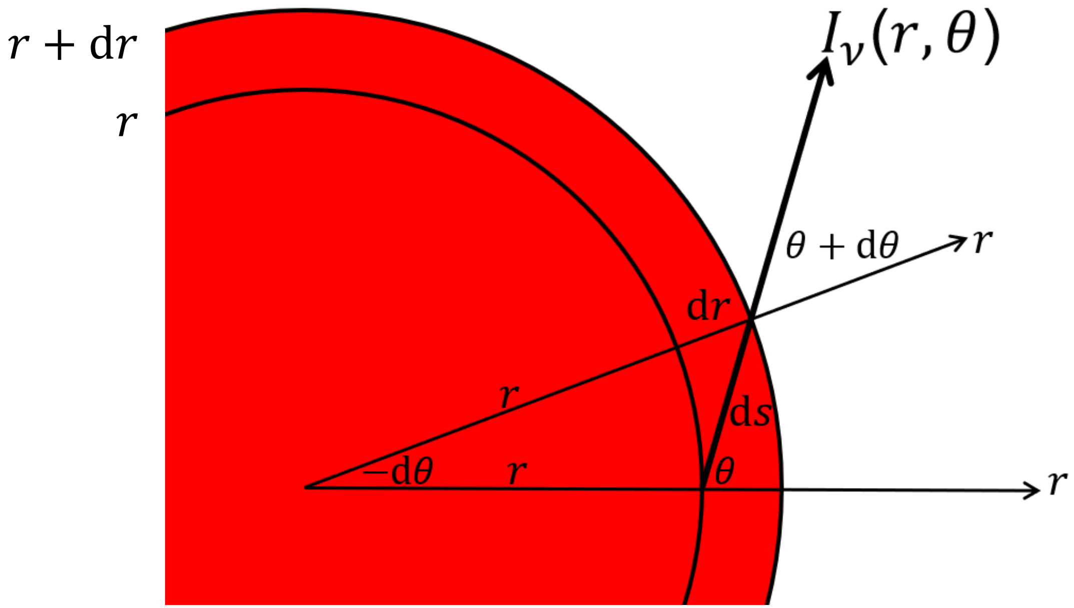


- In novae, plane parallelism is invalid.
  - Instead, considering in spherical geometry is needed
  - θ changes along the ray of light!



Appendix: $\frac{\mathrm{d}}{\mathrm{d}s}$ in the Spherical Geometry + Wind

- Considering total derivative, $\frac{\mathrm{d}}{\mathrm{d}s} = \frac{\mathrm{d}r}{\mathrm{d}s}\frac{\partial}{\partial r} - \frac{\mathrm{d}\theta}{\mathrm{d}s}\frac{\partial}{\partial \theta}$
- From the figure, $\mathrm{d}r = \mathrm{d}s\cos\theta$ and $\mathrm{d}\theta = -\frac{\sin\theta}{r}\mathrm{d}s$
  - So, $\frac{\mathrm{d}r}{\mathrm{d}s} = \cos\theta$ and $\frac{\mathrm{d}\theta}{\mathrm{d}s} = -\frac{\sin\theta}{r}$



- If there is an outward velocity field $\mathrm{v}(r)$,
  - Non-relativistic Doppler shift in frequency:
    - $\mathrm{d}\nu = -\frac{\nu}{c}[\mathrm{v}(r+\mathrm{d}r)\cos(\theta+\mathrm{d}\theta) - \mathrm{v}(r)\cos\theta]$

    $\rightarrow \mathrm{d}\nu \approx -\frac{\nu\mathrm{v}(r)}{cr}\left[\sin^2\theta + \frac{\mathrm{d}\ln\mathrm{v}}{\mathrm{d}\ln r}\cos^2\theta\right]\mathrm{d}s$
  - So, $\frac{\mathrm{d}}{\mathrm{d}s} = \cos\theta\frac{\partial}{\partial r} - \frac{\sin\theta}{r}\frac{\partial}{\partial\theta} - \frac{\nu\mathrm{v}(r)}{cr}\left[\sin^2\theta + \frac{\mathrm{d}\ln\mathrm{v}}{\mathrm{d}\ln r}\cos^2\theta\right]\frac{\partial}{\partial\nu}$

Categories of Spectrum Calculation Codes

- Radiative transfer equation (including scattering in j_ν and α_ν):

$$\mu \frac{\partial I_\nu(r,\mu)}{\partial r} + \frac{1-\mu^2}{r}\frac{\partial I_\nu(r,\mu)}{\partial \mu} - \frac{\nu \mathrm{v}(r)}{rc}\left[\sin^2\theta + \frac{\mathrm{d}\ln \mathrm{v}}{\mathrm{d}\ln r}\cos^2\theta\right]\frac{\partial I_\nu(r,\mu)}{\partial \nu} = j_\nu(r) - \alpha_\nu(r) I_\nu(r,\mu)$$

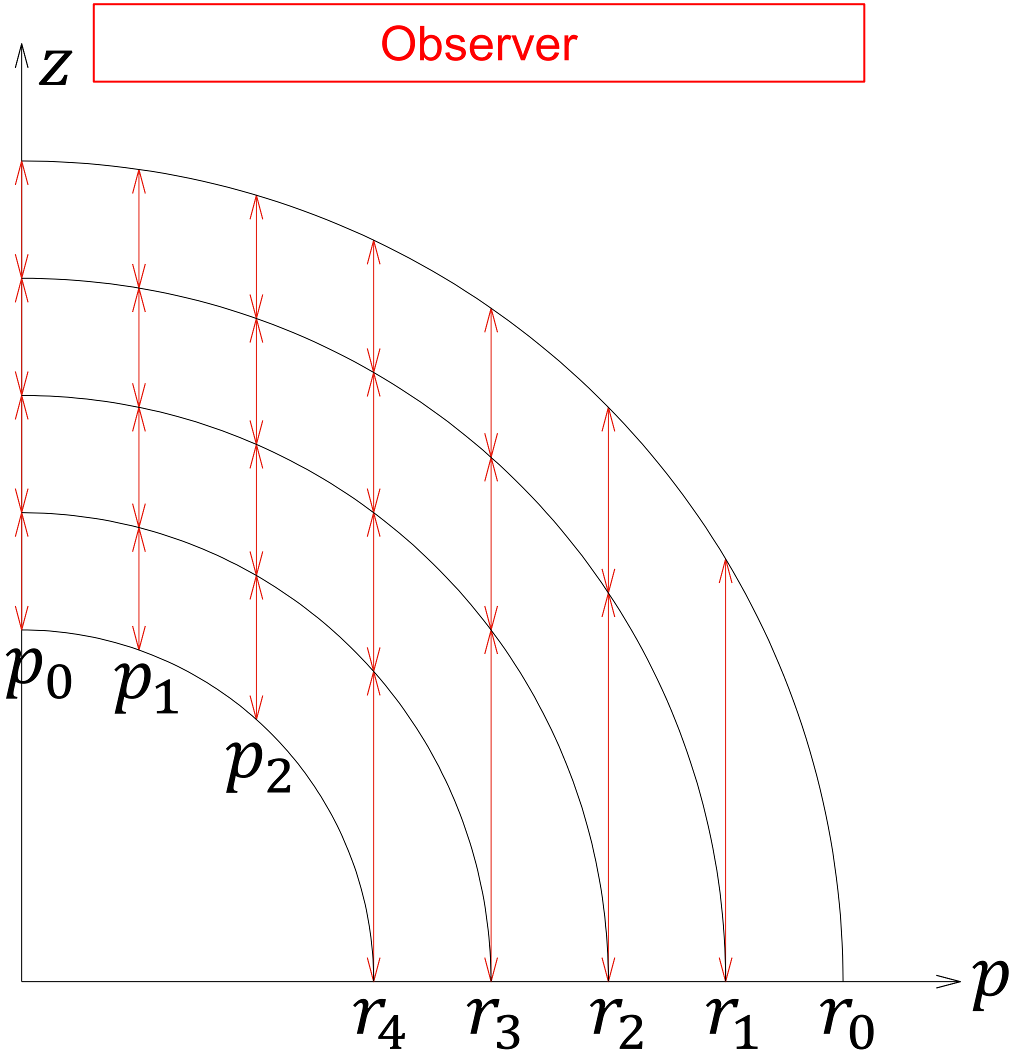
  - For coherent, isotropic scattering, $\alpha_\nu = n_\mathrm{e}\sigma_\mathrm{e}$ and $j_\nu = n_\mathrm{e}\sigma_\mathrm{e} J_\nu$ ($J_\nu = \int I_\nu \frac{\mathrm{d}\Omega}{4\pi}$: mean intensity)

- Monte-Carlo codes are sometimes used.
  - They enable including many lines and calculating line force easier.
  - However, the ionization structure and the source functions are somewhat ad hoc.

- So, we are going to view **difference methods**.
  - CMFGEN, PHOENIX belong to this category.

Impact parameter method

- Solve the radiative transfer (RT) equation in spherical geometry along $p = \mathrm{const}$ rays.
  - For each r, there are many rays.
    - Each ray is different in p (or θ).
  - Observed spectrum is sum of intensities on r_0.



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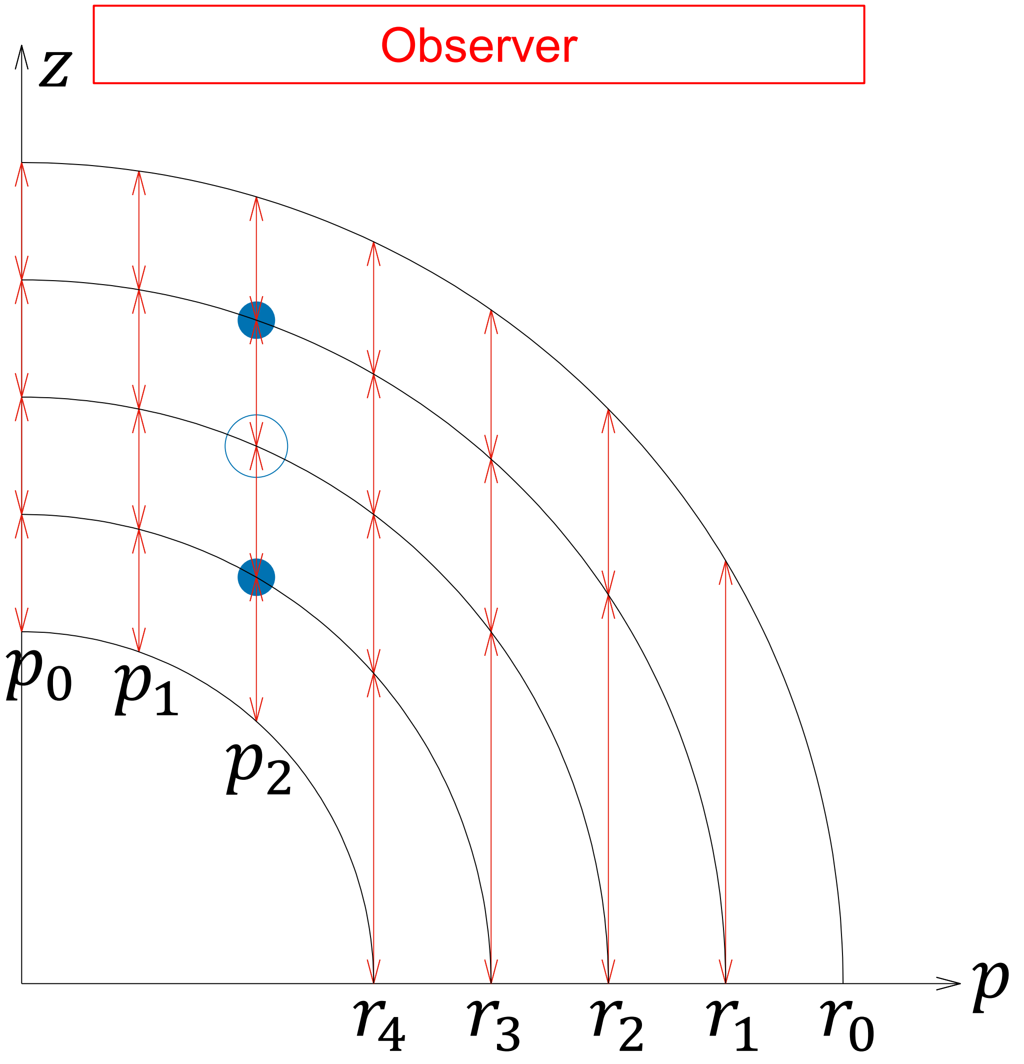
- Without scattering, O depends on ●
  → tridiagonal.



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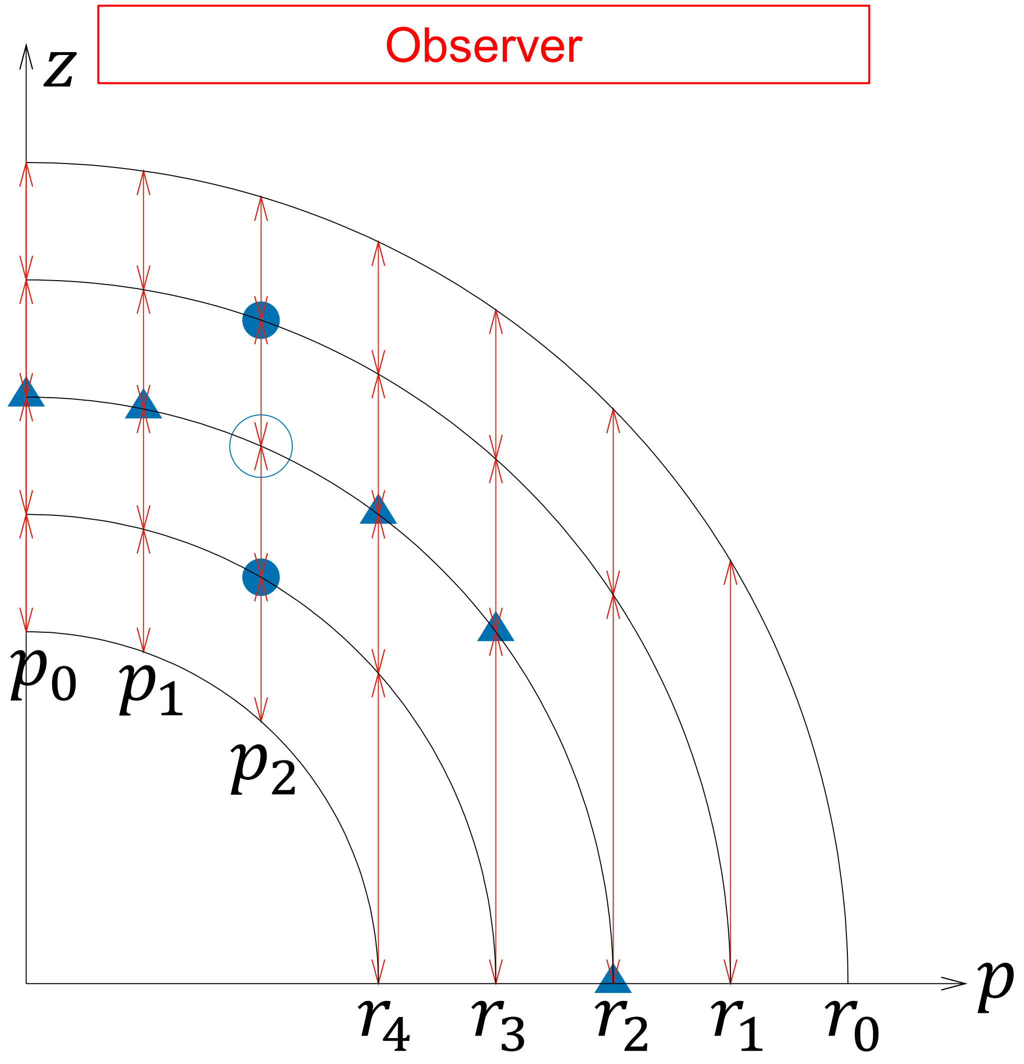
  - For each r, there are many rays.
    - Each ray is different in p (or θ).
  - Observed spectrum is sum of intensities on r_0.
- Without scattering, O depends on ●
  → tridiagonal.
- With scattering, O also depends on ▲
  → difficult to compute!
  (though it's still "tridiagonal" vectoring O and ▲)



Appendix. Feautrier Method

- $I_\nu^{+/-}$: upward/downward specific intensity
- From the transfer equation,
  - $\frac{\partial I_\nu^+}{\partial z} - \frac{\nu \mathrm{v}(r)}{cr}\left(1-\mu^2+\mu^2\frac{\mathrm{d}\ln\mathrm{v}}{\mathrm{d}\ln r}\right)\frac{\partial I_\nu^+}{\partial \nu} = -\alpha_\nu I_\nu^+ + j_\nu$
  - $-\frac{\partial I_\nu^-}{\partial z} - \frac{\nu \mathrm{v}(r)}{cr}\left(1-\mu^2+\mu^2\frac{\mathrm{d}\ln\mathrm{v}}{\mathrm{d}\ln r}\right)\frac{\partial I_\nu^-}{\partial \nu} = -\alpha_\nu I_\nu^- + j_\nu$
- Feautrier variables (Feautrier 1964):
  - $\mathcal{J}_\nu = \frac{I^+ + I^-}{2}$ and $\mathcal{H} = \frac{I^+ - I^-}{2}$

→ equations:

  - $\frac{\partial \mathcal{J}_\nu}{\partial z} = \frac{\nu \mathrm{v}(r)}{cr}\left(1-\mu^2+\mu^2\frac{\mathrm{d}\ln\mathrm{v}}{\mathrm{d}\ln r}\right)\frac{\partial \mathcal{H}_\nu^+}{\partial \nu} - \alpha_\nu \mathcal{H}_\nu$
  - $\frac{\partial \mathcal{H}_\nu}{\partial z} = \frac{\nu \mathrm{v}(r)}{cr}\left(1-\mu^2+\mu^2\frac{\mathrm{d}\ln\mathrm{v}}{\mathrm{d}\ln r}\right)\frac{\partial \mathcal{J}_\nu^+}{\partial \nu} - \alpha_\nu \mathcal{J}_\nu + j_\nu$

Moment Equation

- Radiative transfer equation (including scattering: for coherent):

$$\mu \frac{\partial I_\nu(r,\mu)}{\partial r} + \frac{1-\mu^2}{r}\frac{\partial I_\nu(r,\mu)}{\partial \mu} - \frac{\nu \mathrm{v}(r)}{rc}\left[\sin^2\theta + \frac{\mathrm{d}\ln \mathrm{v}}{\mathrm{d}\ln r}\cos^2\theta\right]\frac{\partial I_\nu(r,\mu)}{\partial \nu} = j_\nu(r) - \alpha_\nu(r) I_\nu(r,\mu)$$

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- 0th and 1st order moment equations:
  - $\frac{1}{r^2}\frac{\partial(r^2 H)}{\partial r} - \frac{\nu v}{rc}\left[\frac{\partial(J-K)}{\partial \nu} + \frac{\mathrm{d}\ln v}{\mathrm{d}\ln r}\frac{\partial K}{\partial \nu}\right] = j_\nu - \alpha_\nu J$
  - $\frac{\partial K}{\partial r} + \frac{3K-J}{r} - \frac{\nu v}{rc}\left[\frac{\partial(H-N)}{\partial \nu} + \frac{\mathrm{d}\ln v}{\mathrm{d}\ln r}\frac{\partial N}{\partial \nu}\right] = -\alpha_\nu H$
  - Here, $[J, H, K, N] = \frac{1}{2}\int_{-1}^{1} I_\nu(r,\mu)[1,\mu,\mu^2,\mu^3]\,\mathrm{d}\mu$

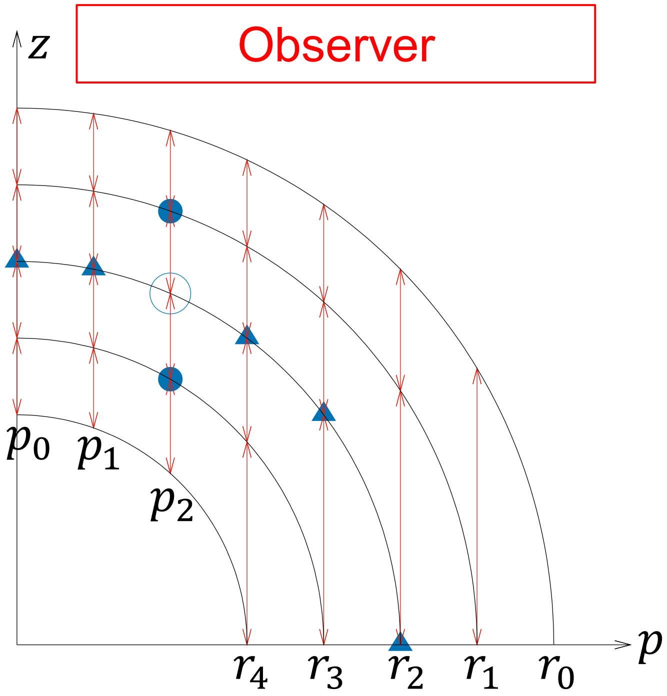
- Eddington Factors
  - $f(r,\nu) = \frac{K(r,\nu)}{J(r,\nu)}$ and $g(r,\nu) = \frac{N(r,\nu)}{H(r,\nu)}$ or $g'(r,\nu) = \frac{N(r,\nu)}{J(r,\nu)}$

Variable Eddington Factors

- 0th and 1st order moment equations:
  - $\frac{1}{r^2}\frac{\partial(r^2 H)}{\partial r} - \frac{\nu v}{rc}\left[\frac{\partial(J-K)}{\partial \nu} + \frac{\mathrm{d}\ln v}{\mathrm{d}\ln r}\frac{\partial K}{\partial \nu}\right] = j_\nu - \alpha_\nu J$
  - $\frac{\partial K}{\partial r} + \frac{3K-J}{r} - \frac{\nu v}{rc}\left[\frac{\partial(H-N)}{\partial \nu} + \frac{\mathrm{d}\ln v}{\mathrm{d}\ln r}\frac{\partial N}{\partial \nu}\right] = -\alpha_\nu H$
    - Here, $[J, H, K, N] = \frac{1}{2}\int_{-1}^{1} I_\nu(r,\mu)[1, \mu, \mu^2, \mu^3]\,\mathrm{d}\mu$ (four unknowns!)
- Eddington factors
  - $f(r,\nu) = \frac{K(r,\nu)}{J(r,\nu)}$ and $g(r,\nu) = \frac{N(r,\nu)}{H(r,\nu)}$ or $g'(r,\nu) = \frac{N(r,\nu)}{J(r,\nu)}$
- Assuming two Eddington factors are known, we can solve J, H, K, N.

  → We can regard the scattering emissivity (e.g., $j_\nu = n_e\sigma_e J_\nu$) "known"
  - Without scattering, O depends only on ● → easy to compute I_ν!

    (From I_ν, we can compute moments J, H, K, N and Eddington factors → iteratable!)



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  - Non-LTE

Non-LTE

- LTE (Local Thermal Equilibrium)
  - The statistical distribution between levels is Canonical (∝ exp (-E/k_BT)) at all (spatial) points (= local).
  - Saha equation between different ionization stages is also available.
  - T decides everything!

- Non-LTE (non-Local Thermal Equilibrium)
  - Low density and high temperature → Photons break Canonical distribution!
    - The effect of **photo-ionization/excitation is not negligible or greater** than that of **collisional ionization/excitation**.
    - So, Boltzmann, Saha, and Kirchhoff are incorrect in non-LTE.

  → The statistical distributions should also be solved!

  (e.g., Detailed balance between two levels: $n_l(B_{lu} J_{ul} + C_{lu}) = n_u(A_{ul} + B_{ul} J_{ul} + C_{ul})$)
    - J_{ul} is also depending on n_l, n_u through the radiation transfer equation!
  - So, the level populations need to be solved simultaneously with the transfer equation!

  (Energy, charge, and total number conservation are also need to be solved)

Newton Scheme for non-LTE

- Physical state vector at $r = r_d$:
  - $\boldsymbol{\psi}_d = \left(T_d^{\mathrm{e}}, n_d^{\mathrm{e}}, n_d^1, n_d^2, \cdots, n_d^{\mathrm{NL}}, J_d^1, J_d^2, \cdots J_d^{\mathrm{NF}}\right)$
    - NL: number of levels, NF: number of frequency points
- General form of constraint equation(s) at $r = r_d$:
  - $\boldsymbol{P}_d(\boldsymbol{\psi}_d) = 0$
- Iterative calculation:
  - (Exact solution $\boldsymbol{\psi}_d$) = (Nearby, imperfect solution $\boldsymbol{\psi}_d^0$) + (Correction $\delta\boldsymbol{\psi}_d$)

  → Taylor expansion up to 1st order: $\sum_j \frac{\partial \boldsymbol{P}_d}{\partial \psi_{d,j}} \delta\psi_{d,j} = -\boldsymbol{P}_d\left(\boldsymbol{\psi}_d^0\right)$

Radiative Transfer to simplify non-LTE

- Considering removing δJ_d^f from $\sum_j \frac{\partial \boldsymbol{P}_d}{\partial \psi_{d,j}} \delta\psi_{d,j} = -\boldsymbol{P}_d(\boldsymbol{\psi}_d^0)$
  - From radiative transfer equation, $\delta J_d^f = \sum_{d'=1}^{\mathrm{ND}} \left(\sum_{l=1}^{\mathrm{NL}} \frac{\partial J_d^f}{\partial n_{d'}^l} \delta n_{d'}^l + \frac{\partial J_d^f}{\partial n_{d'}^{\mathrm{e}}} \delta n_{d'}^{\mathrm{e}} + \frac{\partial J_d^f}{\partial T_{d'}^{\mathrm{e}}} \delta T_{d'}^{\mathrm{e}} \right)$
  - In practical, CMFGEN uses $d' = d-1, d, d+1$ to decrease the computation.
    - Here, J_d^f is determined by the same inverse matrix as VEF equation.