Nonaxisymmetric Magnetorotational instability in Proto-neutron stars

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Outline

1. Introduction
2. Motivation, Method & Dispersion relation
3. General feature of Dispersion relation
4. Application to the Proto-neutron stars
5. Summary & Discussion

Introduction

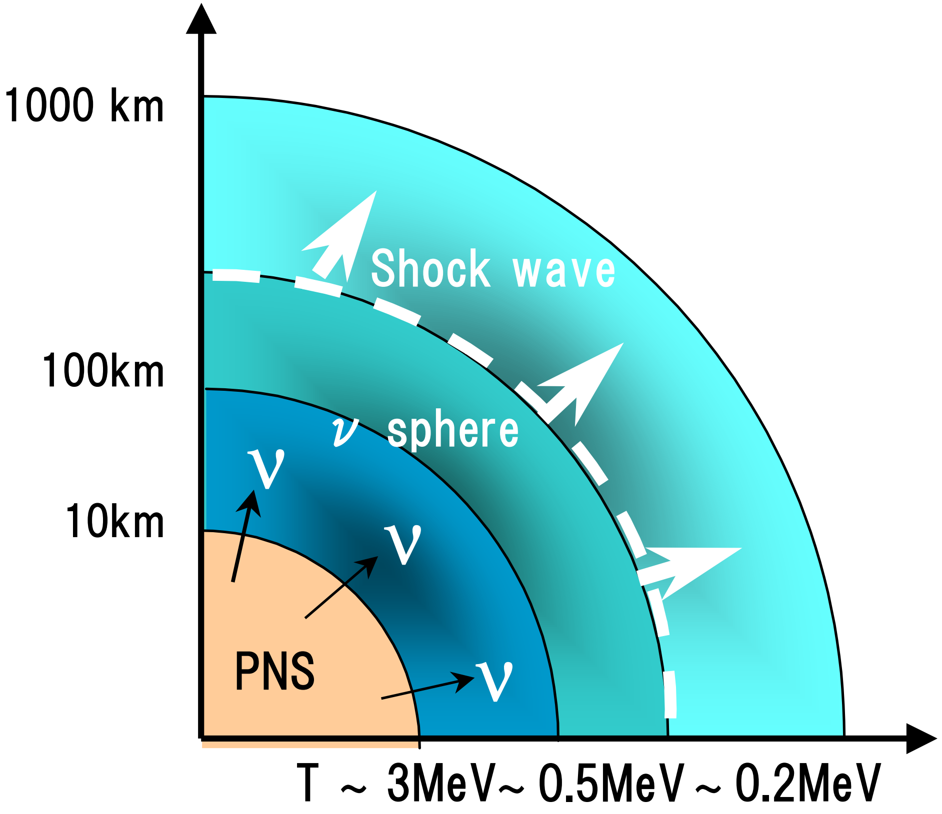
①.Core-collapse supernova

★ Core-collapse supernova

- Explosion phenomena of Massive stars in their final evolutionary phase
- Explosion mechanism is not revealed ⇒ Delayed explosion scenario

★ Delayed explosion scenario

①. Core-collapse of Fe core
②. Core bounce
③. Shock generation
④. Shock heating → decomposition
ν production
⑤. Shock stalled (Accreting shock)
⑥. ν radiation from PNS
(born behind shock wave)
⑦. Shock acceleration by ν heating
⑧.Accelerated shock leads to SNe



②.Current situation

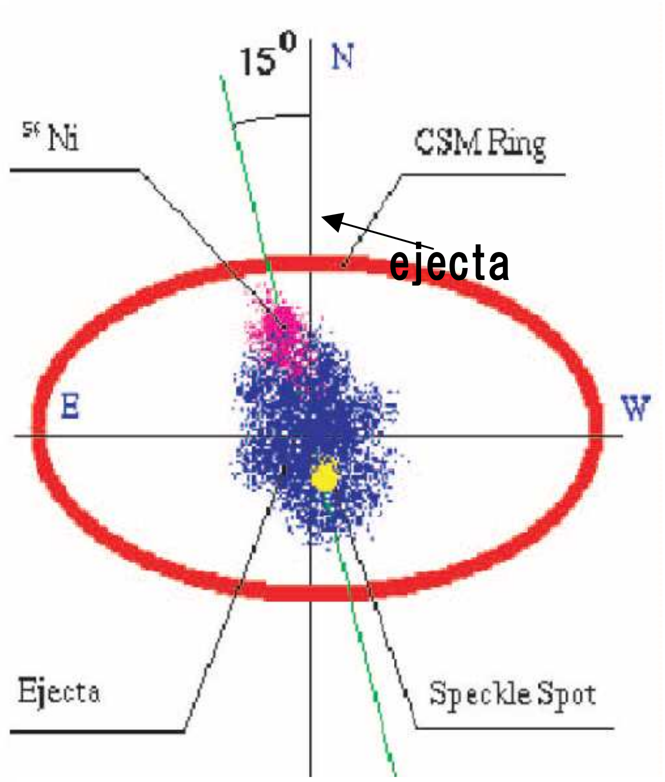
★ 1D-spherical simulation（GR、Boltzmann eq for ν transport）

⇒ Most simulations fail to produce explosion (cf. Liebendorfer et al. 2003)

★ Observational suggestions

⇒ Asymmetric structure of SN1987A (Wang et al. 2002)

Multi-dimensional effects
（convection、rotation、magnetic fields）



Aspherical structure of SN1987A

★ Convection in proto-neutron stars

- amplification of the neutrino luminosity
- Assistance of the delayed explosion
- 2-D axisymmetric simulations (cf. Janka&Muller 1996; Buras et al. 2003)

③.Simulation of Buras et al.2003

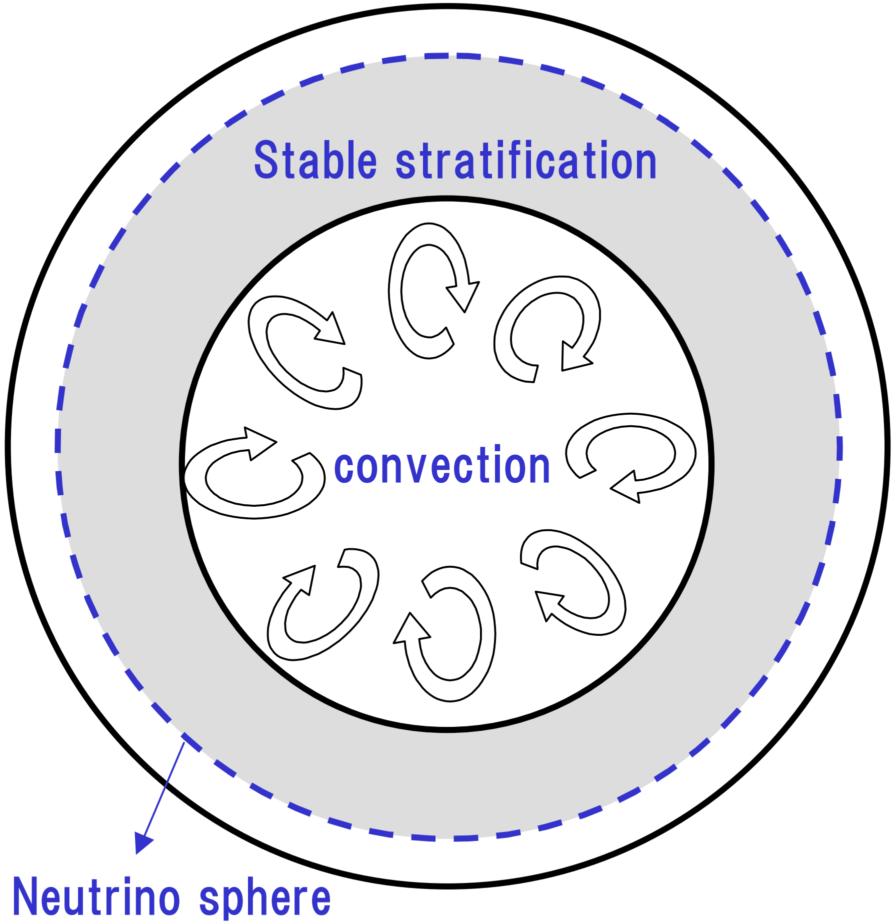


FIG: Schematic picture of
internal structure of PNS

- ★ Convective core is surrounded by a convectively stable shell
- ★ Convection in PNSs has been to have little influence on the emission of neutrino.
- ★ It is irrelevant for SN dynamics.

SNe is not recreated numerically even if we consider 2D effects

Motivation, Method & Dispersion relation

Motivation of our research

★ The physics included in existing researches can’t lead to SNe
⇒ Some additional physics lack in current situation.

- I focus on the stably stratified envelope of PNS.
- Non-axisymmetric MHD instabilities (3-D effects) due to rotation & magnetic fields is studied.

★ Expected effects in MHD instabilities

①. Angular momentum transport due to magnetic turbulence

- Aspherical convection due to rigid rotation (Miralles et al.2004)
- Amplification of polar neutrino luminosities preferentially??

②. Amplification of magnetic field (Balbus&Hawley1991, Spruit 1999)

- Mixing by magnetoconvection ⇒ Amplification of neutrino flux ??

③. Matter heating by magnetic reconnection (Socrates 2005)

- Amplification of neutrino luminosities ??

Method - Local linear analysis -

★Basic equation – Ideal MHD equation –

$$\frac{\partial \rho}{\partial t} = -\nabla \cdot (\rho \boldsymbol{u})$$

$$\frac{\mathrm{d}\boldsymbol{u}}{\mathrm{d}t} = -\frac{1}{\rho}\nabla\left(p + \frac{1}{2\mu}\boldsymbol{B}^2\right) + \boldsymbol{B}\cdot\nabla\boldsymbol{B} + \boldsymbol{g}$$

$$\frac{\partial \boldsymbol{B}}{\partial t} = \nabla \times (\boldsymbol{u} \times \boldsymbol{B})$$

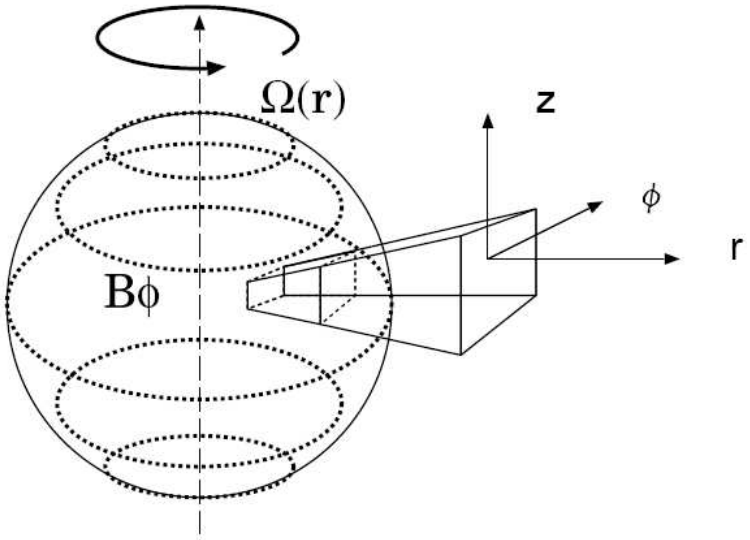
$$\frac{\mathrm{dS}}{\mathrm{d}t} = 0$$

★Local linear analysis（Boussinesq approx、）

★ hydrostatic $\frac{B_\phi^2}{\mu\rho r} + \frac{1}{\rho}\frac{\partial p}{\partial r} + \frac{B_\phi}{\mu\rho}\frac{\partial B_\phi}{\partial r} = -g_r + r\Omega^2$

★normal mode(δ $\propto \exp[i(k_r r \underline{+ m\phi +} k_z z - \omega t)]$)

★cylindrical coordinate



★angular velocity: $\Omega \propto r^{-q}$

★initial condition

$\mathbf{u} = (0, r\Omega(r), 0)$

$\mathbf{B} = (0, B_\varphi(r), 0)$

$\rho = \rho(r,z)$

$p = p(r,z)$

Dispersion relation

$$\frac{k^2}{k_z^2}\omega^4 - \left[2(1+s)\omega_A^2 + \kappa^2 + 2\frac{k^2}{k_z^2}m^2\omega_A^2 + N_e^2\right]\omega^2 - 8m\omega_A^2\Omega\omega$$
$$-2qm^2\omega_A^2\Omega^2 - 2(1-s)m^2\omega_A^4 + \frac{k^2}{k_z^2}m^4\omega_A^4 + m^2\omega_A^2N_e^2 = 0\ ,$$

$$k^2 = k_r^2 + k_z^2\ ,$$

$$\kappa^2 = 2(2-q)\Omega^2 = \frac{1}{r^3}\frac{\partial(r^2\Omega)^2}{\partial r}\ ,$$:Epicyclic frequency

$$\omega_A = \frac{B_\phi}{\sqrt{\mu\rho}r} = \frac{v_{A\phi}}{r}\ ,$$:Alfven frequency

$$N_e^2 = \left(N_r - \frac{k_r}{k_z}N_z\right)^2\ .$$:Effective Brunt-Vaisala frequency

$$N^2 = N_T^2 + N_{Y_l}^2\ ,$$

$$N_T^2 = -\frac{\alpha \boldsymbol{g}^*}{T}\cdot\left[\left(\frac{\partial T}{\partial p}\right)_{s,Y_l}\nabla p - \nabla T\right]\ ,$$:Thermal Brunt-Vaisala frequency

$$N_{Y_l}^2 = \boldsymbol{g}^*\cdot\xi\nabla Y_l\ .$$:Leptonic Brunt-Vaisala frequency

Important parameters

In present work, there are five important parameters

- ✓ shear parameter “ q ”
- ✓ Brunt-Vaisala frequency “ N ”
- ✓ angular velocity “ Ω ”
- ✓ Alfven frequency “ ω_A ”

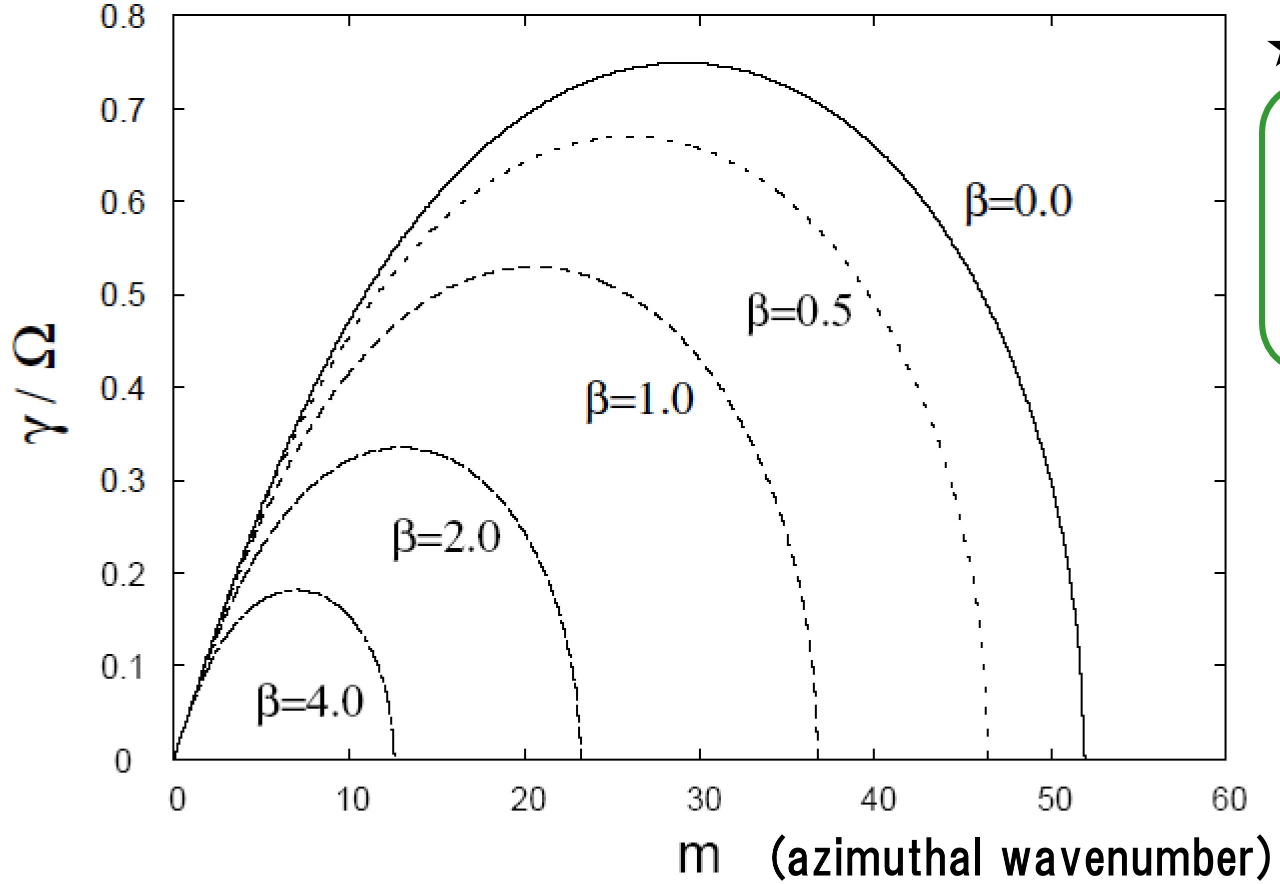
To treat in a general way, we use N/Ω, ω_A/Ω (>>1).
(Typically, in stellar interior, $N \gg \Omega \gg \omega_A$ is satisfied.)

- ✓ The ratio of poloidal wavenumber “ $\beta \equiv k_r/k_z$ ”
- ✓ The azimuthal wavenumber “ m ”

General features of the dispersion relation

Result1. β-dependence

Back to 17



★parameters

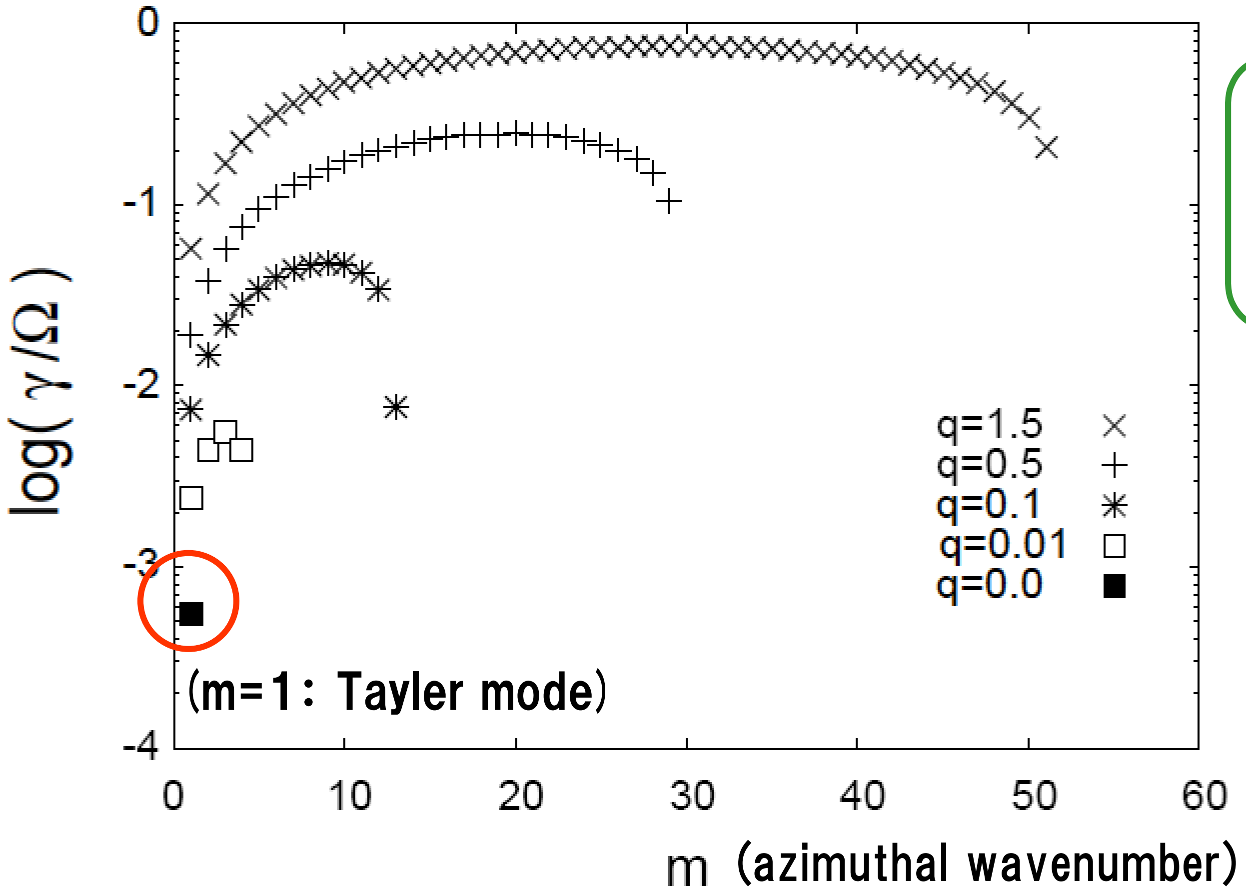
- $N/\Omega = 0.0$
- $q = 1.5$
- $\Omega/\omega_A = 30$

★ The modes with smaller β grow faster when m is fixed.

★ The growth rate of maximum growing mode is the order of Ω when $q \sim O(1)$

★ This instability is known as Non-axisymmetric MRI (NMRI) (Kim&Ostriker2000).

Result2. q-dependence



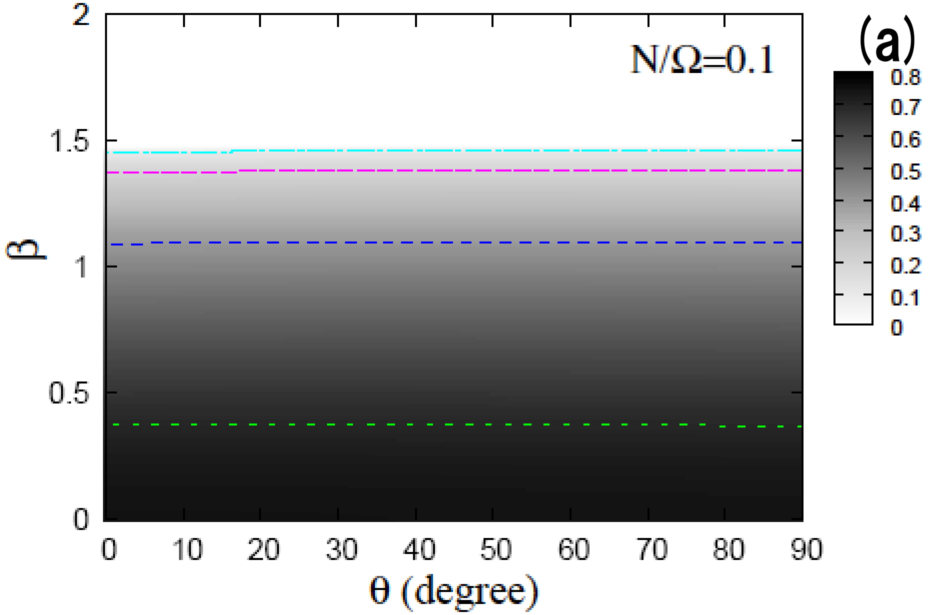
★parameter

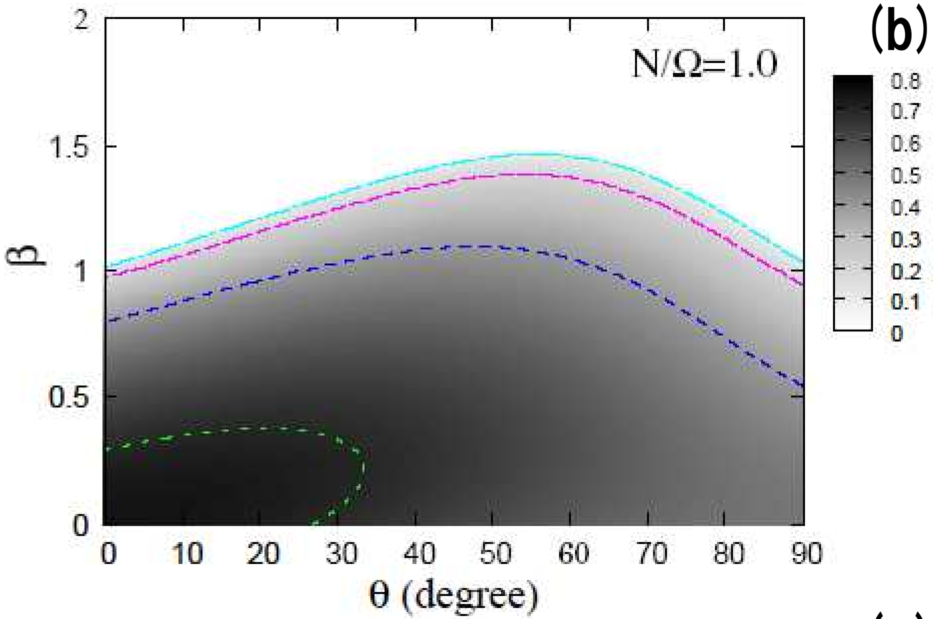
- β = 0.0
- N = 0.0
- Ω / ω_A=30

★ Growth rate decrease with decreasing “q”

★ NMRI reduce to the Tayler instability at the limit of rigid rotation
(⇒ Tayler-Spruit dynamo :Acheson 1978 ; Spruit1999).

Result3. Stable stratification：NMRI modes (m=29)





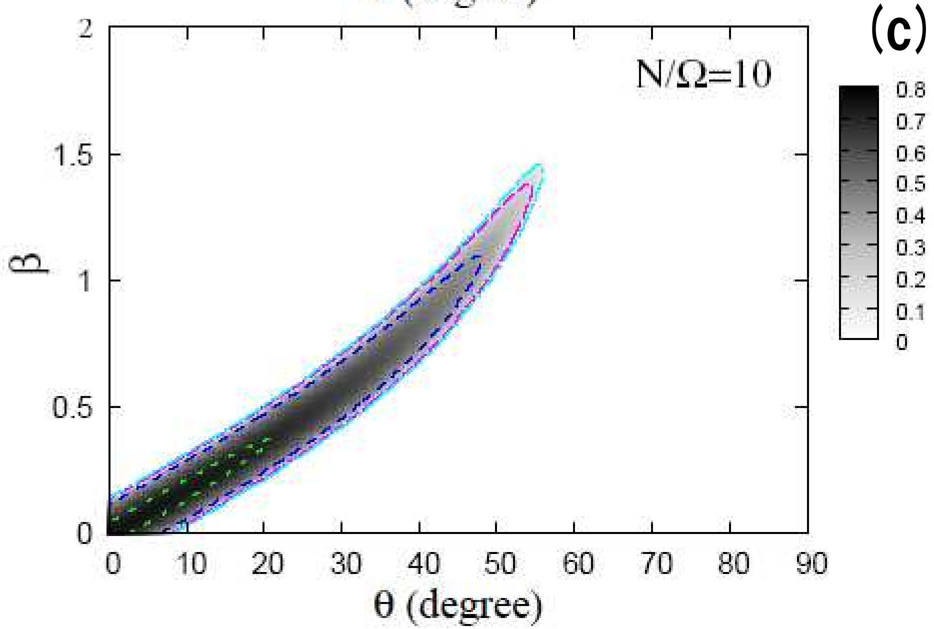
★ Basically, the growth rate is suppressed with increasing N

★ The mode with β=kr/kz ～ Nr/Nz is not suppressed and remained.

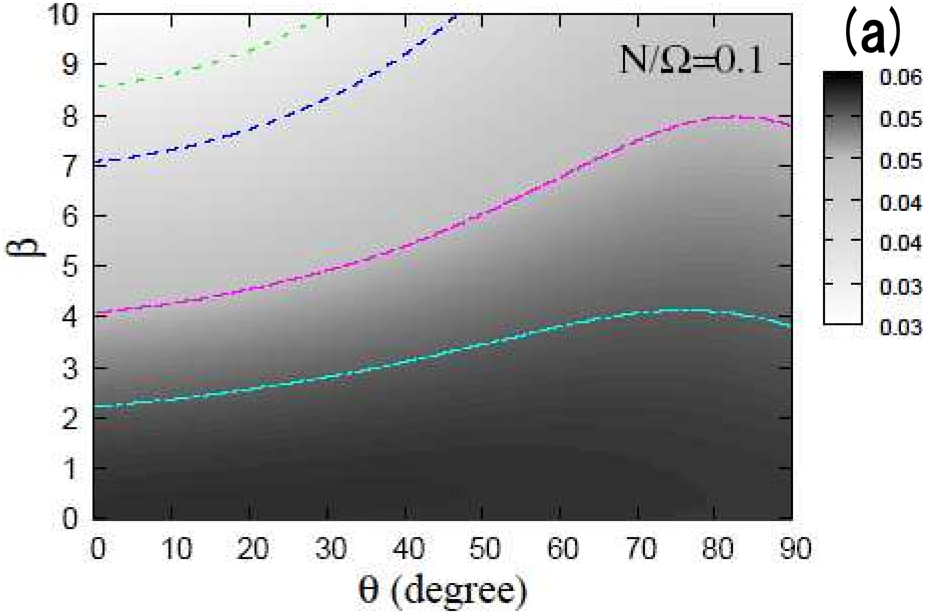
★ The mode with m=29 is suppressed at θ～60° in the case of N>>Ω.

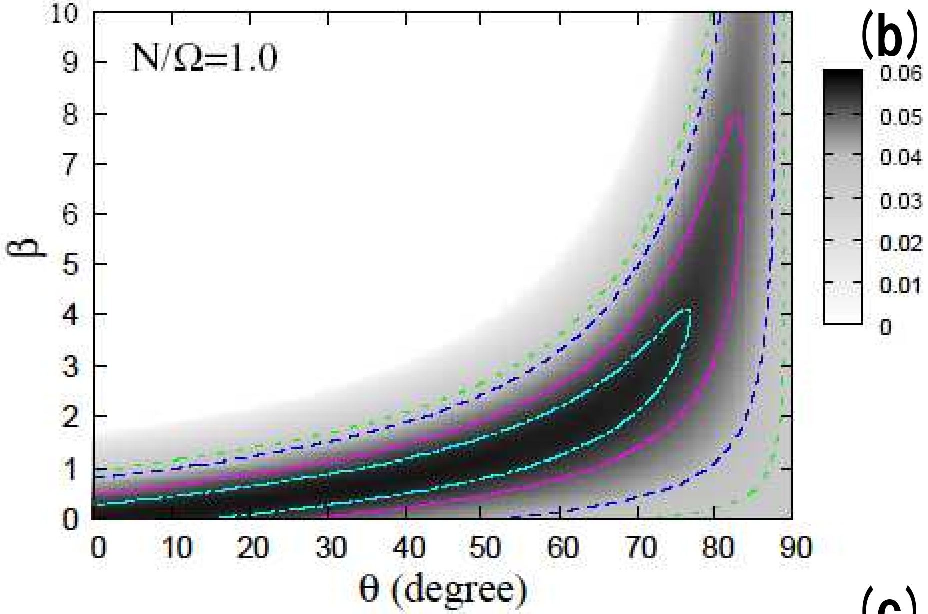
★parameter

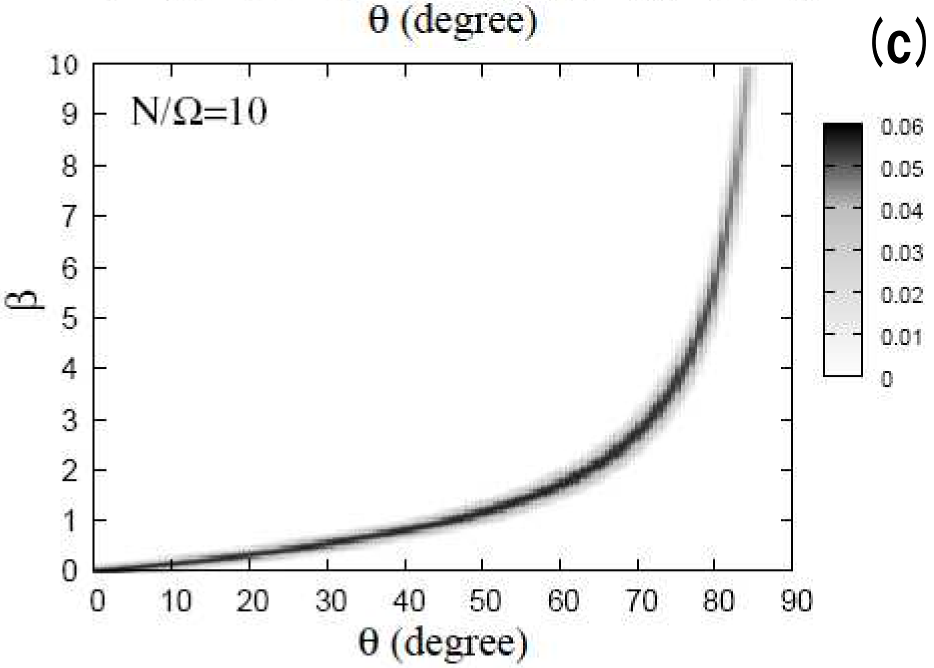
•q = 1.5　•Ω/ω$_A$=30



Result3. Stable stratification：Tayler modes (m=1)


(a)


(b)


(c)

★ Basically, the growth rate is suppressed with increasing N.

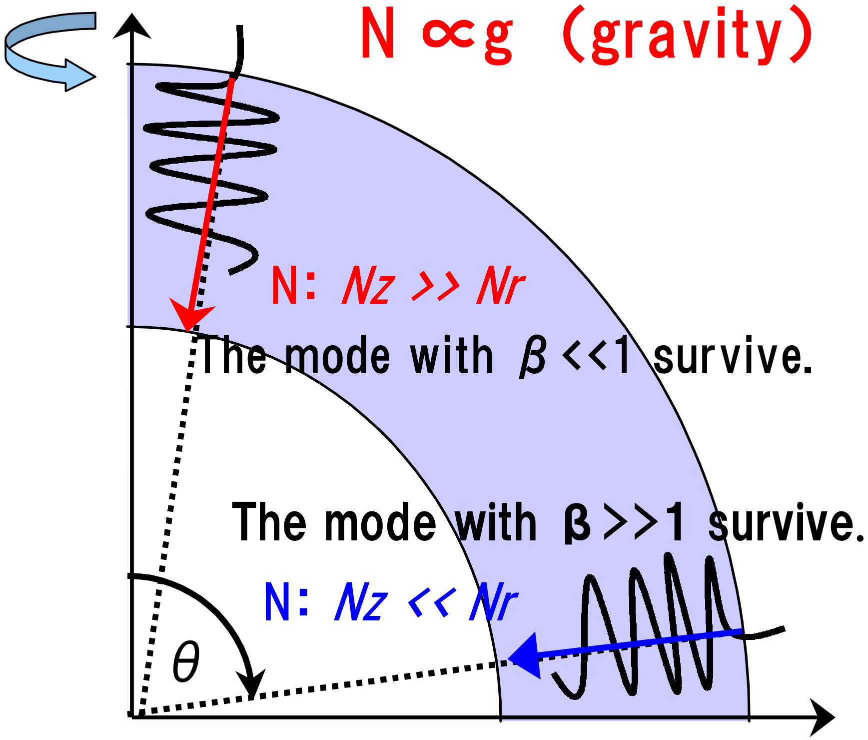
★ The mode with β=kr/kz ～ Nr/Nz is not suppressed and remained.

★ Tayler mode is not suppressed until θ～90° in the case of N >> Ω.

★parameter

•q = 1.5 •Ω/ω_A=30

Result3. Stable stratification:Interpretation



N ∝g (gravity)

N: *Nz* >> *Nr*

The mode with β<<1 survive.

The mode with β>>1 survive.

N: *Nz* << *Nr*

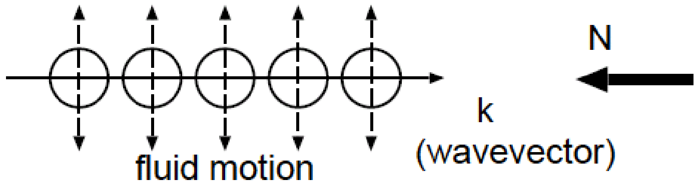
Equatorial plane (E.P)

$$\theta \equiv \tan^{-1}(r/z),$$

$$N_r = N\sin\theta, \quad N_z = N\cos\theta\ .$$

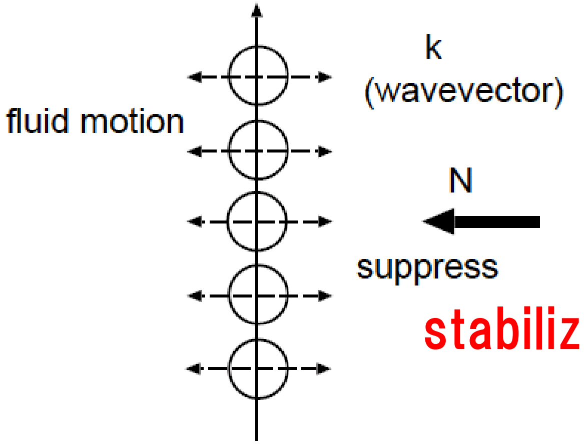
Incompressible approximation
⇒ transverse wave

(a). k is parallele to N



Not stabilized

(a). k is perpendicular to N



stabilized

$$\beta = k_r / k_z \sim N_r / N_z$$

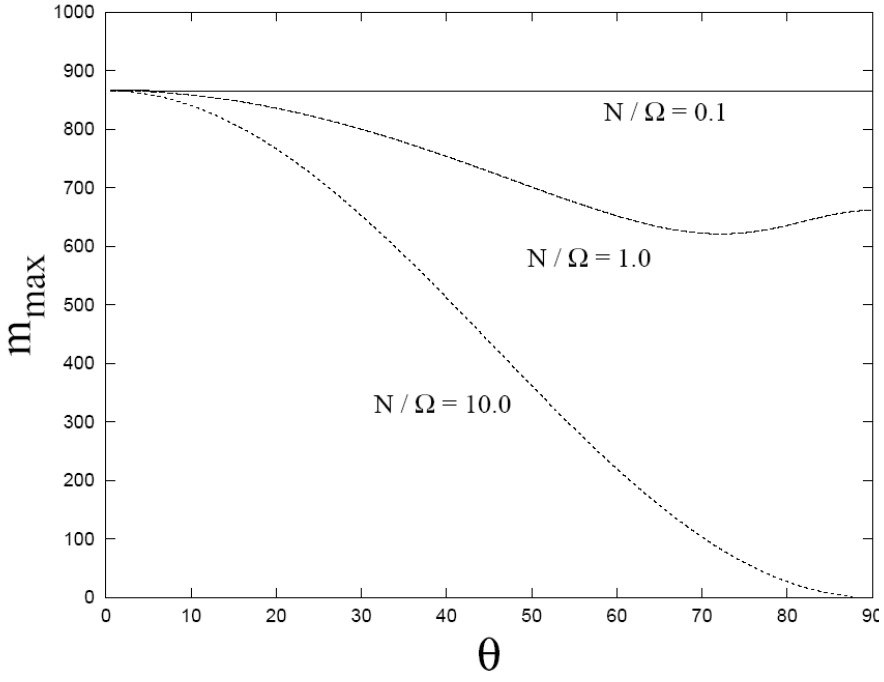
Back to figure1

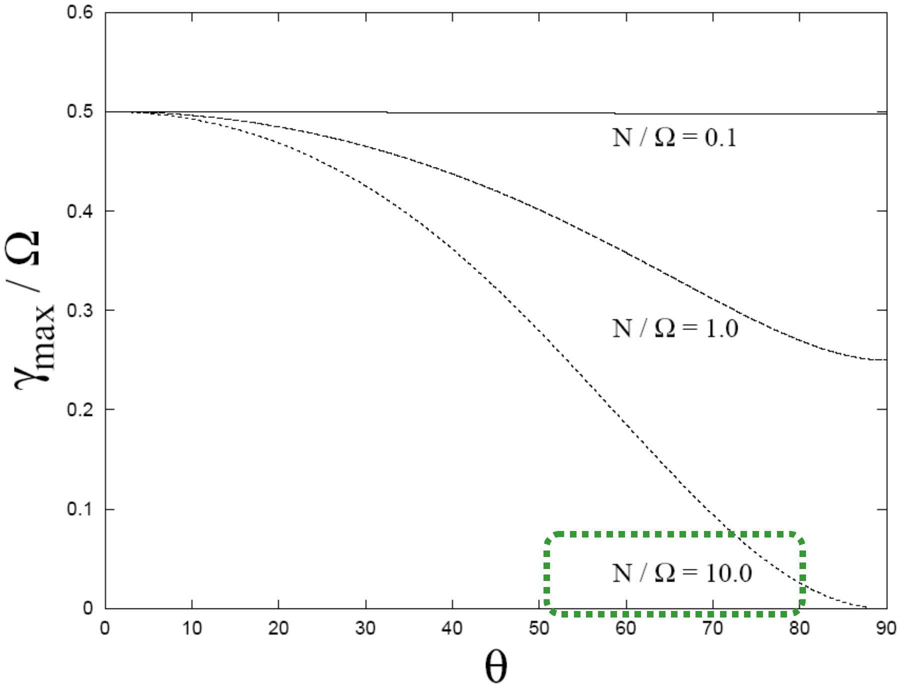
Application to the Proto-neutron stars

Growth rate in the outer envelope of PNS

★ θ dependence of γ_{max}. (The growth rate of most growing mode)

（N is free parameter、q～1、Ω / ω_A =30 （ω_A～1.0 [/sec] ） are fixed ）





★ Maximum growth rate is suppressed with increasing N/Ω.

★ The mode which have maximum growth rate （m_{max} ） have same properties.

★ In almost all region, NMRI grows and Tayler mode is dominant just around E.P

Summary & Discussion

Summary

★ NMRI reduce to the Tayler instability at the limit of rigid rotation.

★ NMRI grows in stably stratified envelope of PNS.

★ The maximum growth rates depend on the size of N/Ω.

- The growth rate is suppressed from E.P. with increasing N/Ω.
- When N>>Ω, NMRI grows preferentially in rotational axis.

Discussion - Expected magnetic fields -

◎ Upper limit of amplification of magnetic fields？？？？

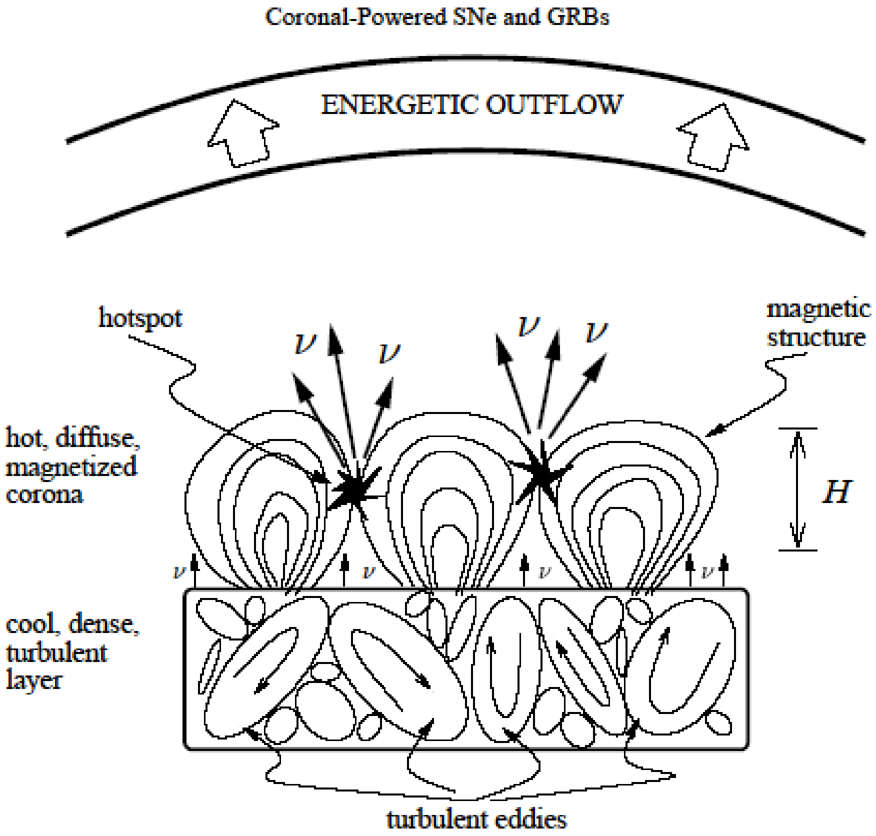
- We apply the results known in accretion disk to PNS,

⇒ magnetic energy saturate at 1−10% of thermal one（Sano&Inutsuka2001）

- Typical temperature of PNS ～3Mev ⇒ 10^{52}erg (Thermal energy)
- Corresponding magnetic fields is 10^{15-16}gauss

Discussion - Nonlinear growth of NMRI -

★ In stably stratified region, there might be additional mixing and heating process which caused by magnetoconvection of magnetic flux tube generated by MHD instabilities.



(Ramirez-Ruiz&Socrates2005)

■ magnetic turbulence

⇒ viscous energy dissipation

⇒ Transformation of rotational energy to thermal energy.

⇒ Amplification of neutrino luminosity

■ Amplification of magnetic fields

⇒ ・Magnetoconvection

・Magnetic reconnections

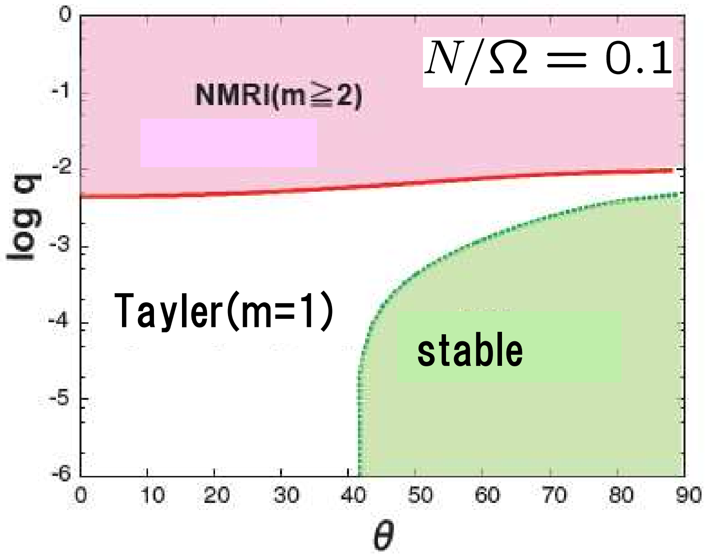
⇒ Amplification of neutrino flux

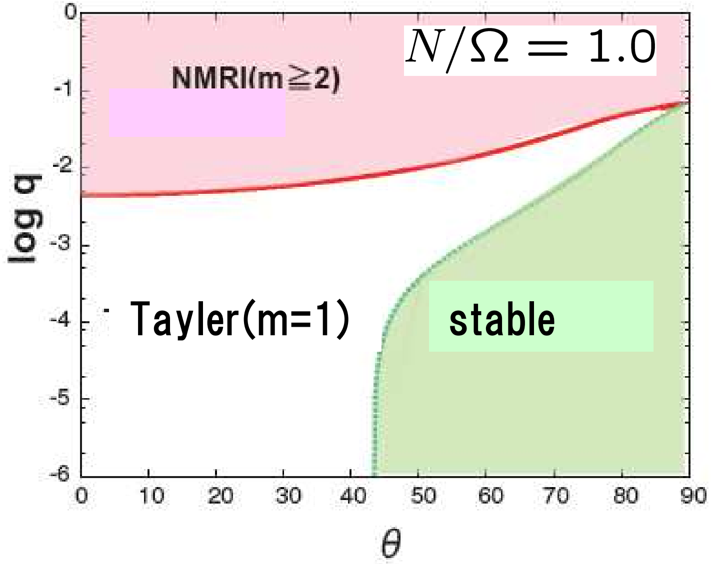
To understand more quantitatively, we must perform 3D MHD simulation.

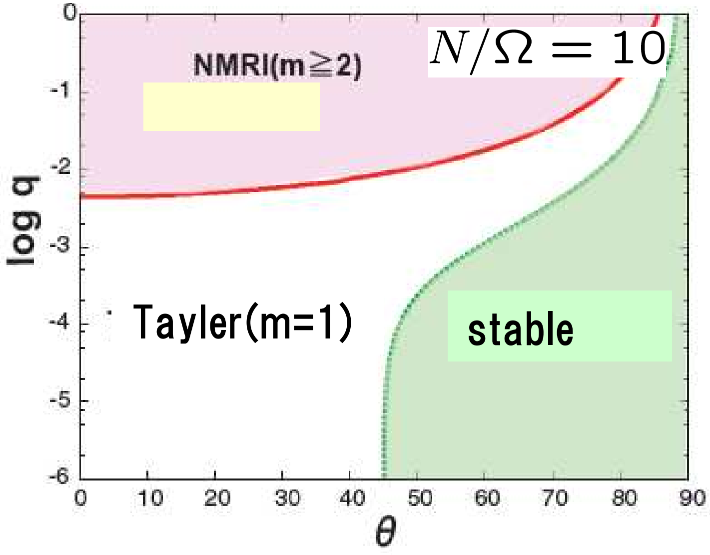
Please speak very slowly and clearly.

Discussion

★ Dependence of Brunt-Vaisala frequency N (preliminary for MRI).





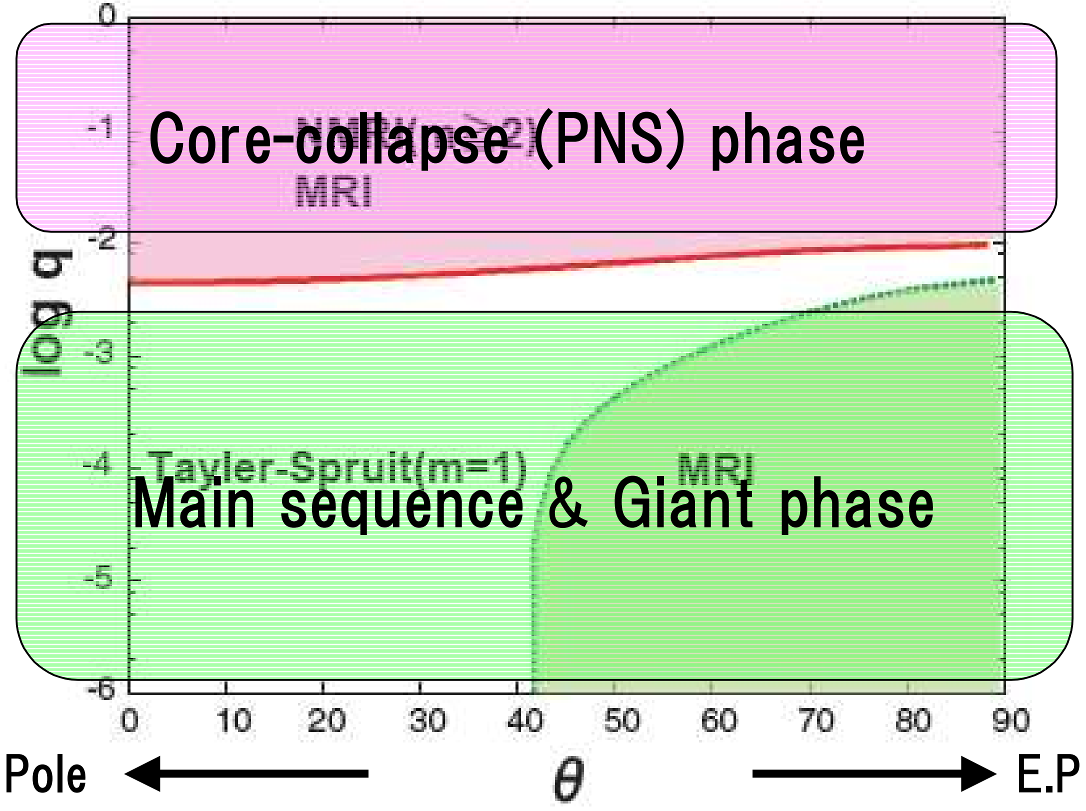


★ When q is larger than q_{crit}, then NMRI is dominant over the Tayler instability.

★ The properties of schematic diagram is hardly changed by the size of N.

Discussion

Correspondence of evolution phase of massive stars to the shear parameter.



Discussion

★ Physical meaning of qcrit (& θ crit) for determination of the dominant mode in various systems.

(i). qcrit

- Typical growth rate of NMRI : qΩ
- Typical growth rate of Tayler instability : ω_A/Ω

$q\Omega \sim \omega_A^2/\Omega$ ➡ $q_{crit} \sim (\omega_A/\Omega)^2 \sim 10^{-3}$

(ii). θ crit (preliminary)

- stability criterion for Tayler instability

$$s > \frac{m^2}{2} - 1 + q\left(\frac{\Omega}{\omega_A}\right)^2 + \frac{1}{2}\frac{\eta}{\kappa}\left(\frac{N_e}{\omega_A}\right)^2$$

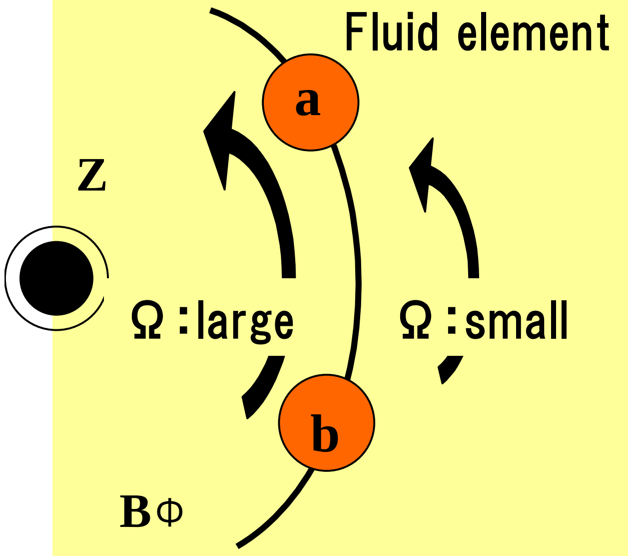
Substitute appropriate parameter ➡ $\tan\theta \gtrsim 1$

$\theta \gtrsim 45^\circ$

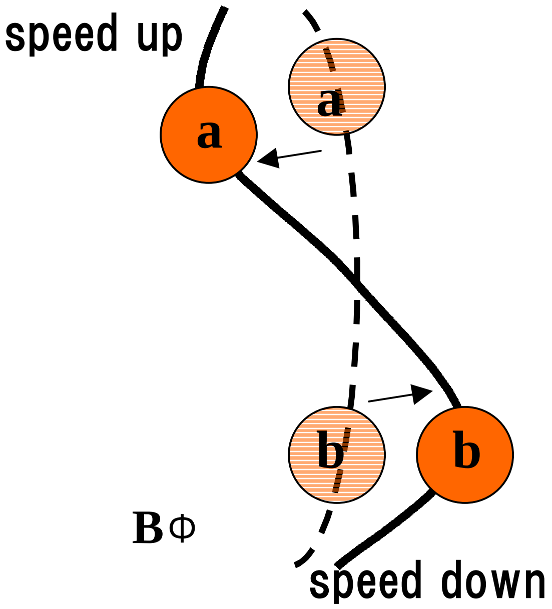
Mechanism of Nonaxisymmetric magnetorotational instability

① cylindrical coordinate (r,ϕ,z).We consider a toroidal magnetic field. Two fluid elements are frozen in field line.
Ωin > Ωout

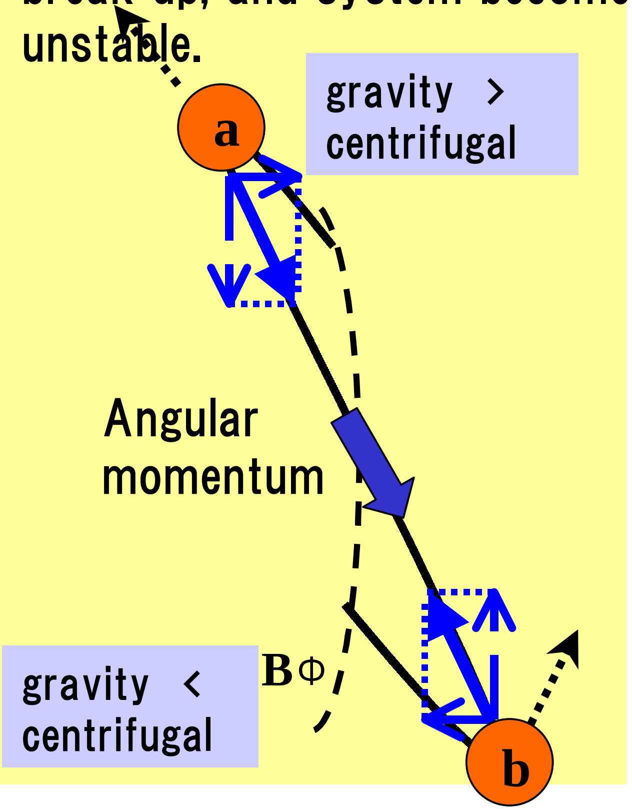
gravity = centrifugal force



② Two fluid elements are perturbed. Angular momentum is conserved in this time. Fluid element a become faster、Fluid element b become slower.



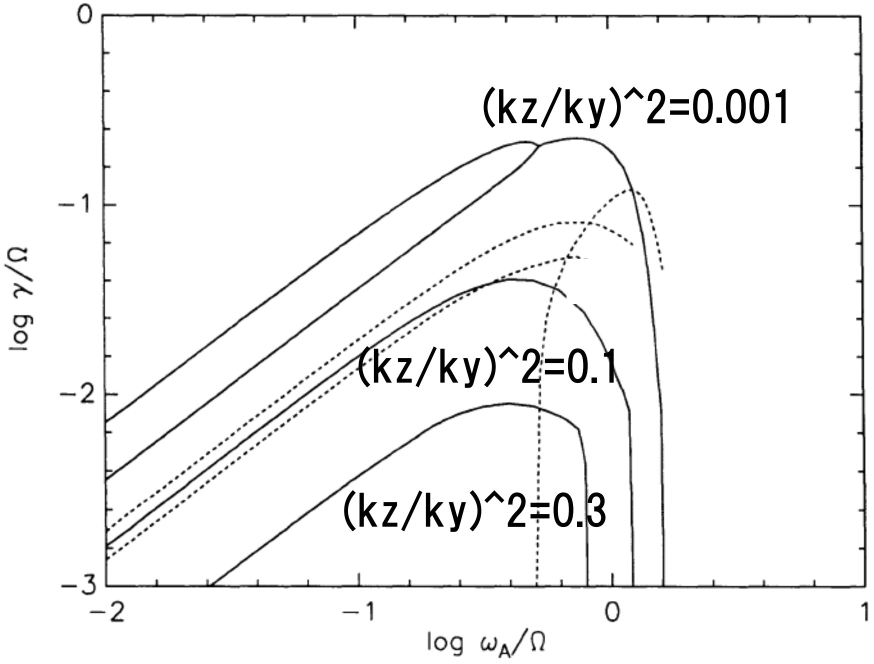
③ Field line have the spring effects, angular momentum is transported with acting torque between two fluid elements. Balance of force break up, and system become unstable.



(EX). Effectiveness of our dispersion relation

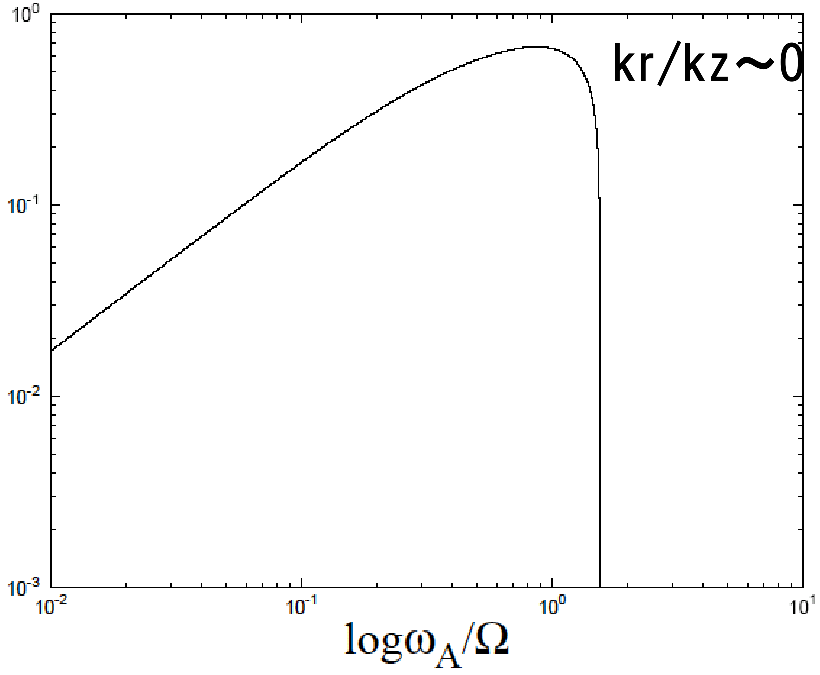
Matsumoto&Tajima1995

Radial structure(＋initial value problem)



Our dispersion relation

(Normal mode analysis)



- The dependence of wavenumber is qualitatively same each other.

  ⇒ So, we can treat local approximation.
- Normal mode analysis ⇒ I can choice parameters freely.

Abstract

Motivation

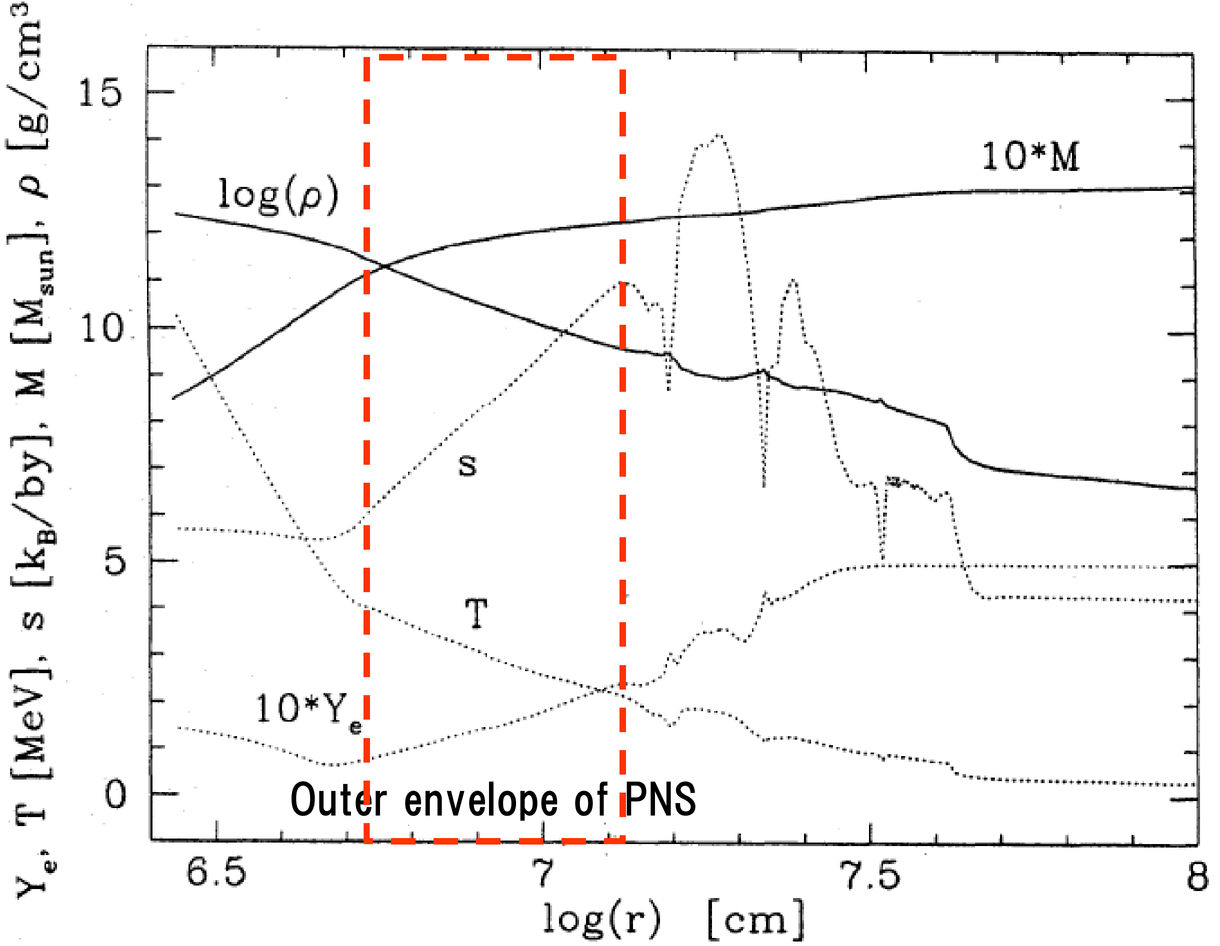
- Revealing the mechanism for core-collapse SNe.
- Magneto - rotational effects on core-collapse SNe.
- MHD instabilities in stably stratified regions of Proto-neutron stars (PNSs).

Results

- In PNSs, NMRI is more important than Tayler instability.
- NMRI grows in almost all regions of PNSs.
- Growth rate become larger in rotational axis.

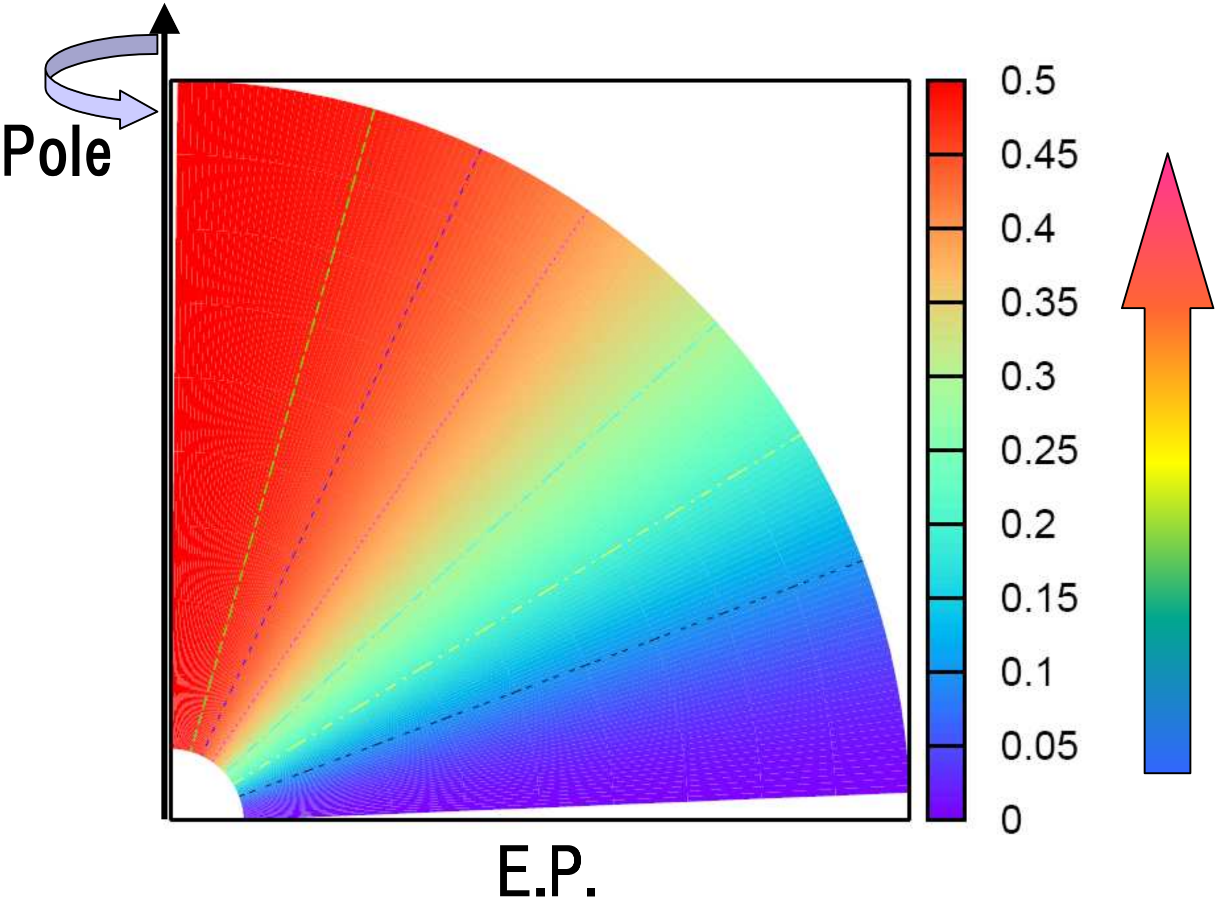
(Supplement) Leptonic gradient

Internal structure of SNe at 50msec after core bounce（Janka＆Muller1996）



★ Leptonic gradient also stabilize the displacement of fluid eleme nts.

Maximum growth rate in the envelope of PNS



- In almost all region of PNS, MHD instabilities grows.
- Growth time is typically the order of O(1) [msec]. It is shorter than the timescale of cooling of proto-neutron stars.
- MHD instabilities grows preferentially around polar axis.

(Ex). Stable stratification

★ Comparison of growth rates of Tayler mode ($m=1$) & NMRI modes ($m \geqq 2$).

★ Basic properties (Masada et al.2005, submitted to Apj)

- The modes with larger “ m ” are more stabilized by a restoring force.
- However, only the mode with “ k // N ” are not suppressed.

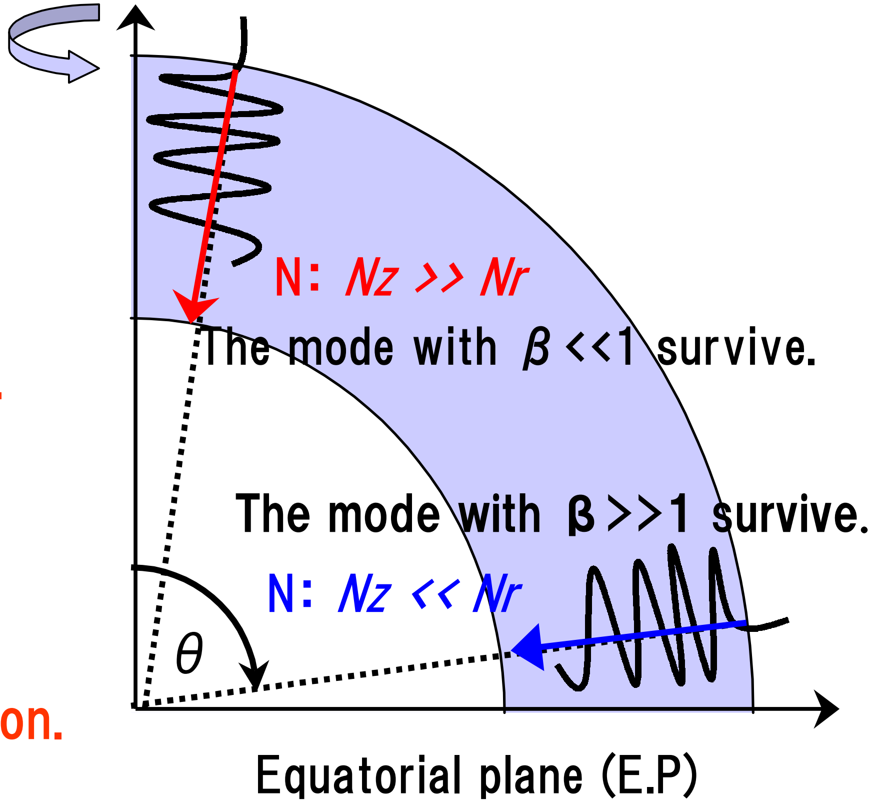
considering fixed strong N ・・・・

①. “q～O(1)” case

- NMRI modes are dominant around pole.
- Tayler mode dominant around E.P.

②. “ q<<1 ” case

- NMRI modes are not unstable.
- Tayler mode only survive in whole region.



Next challenge – 3D MHD simulation -

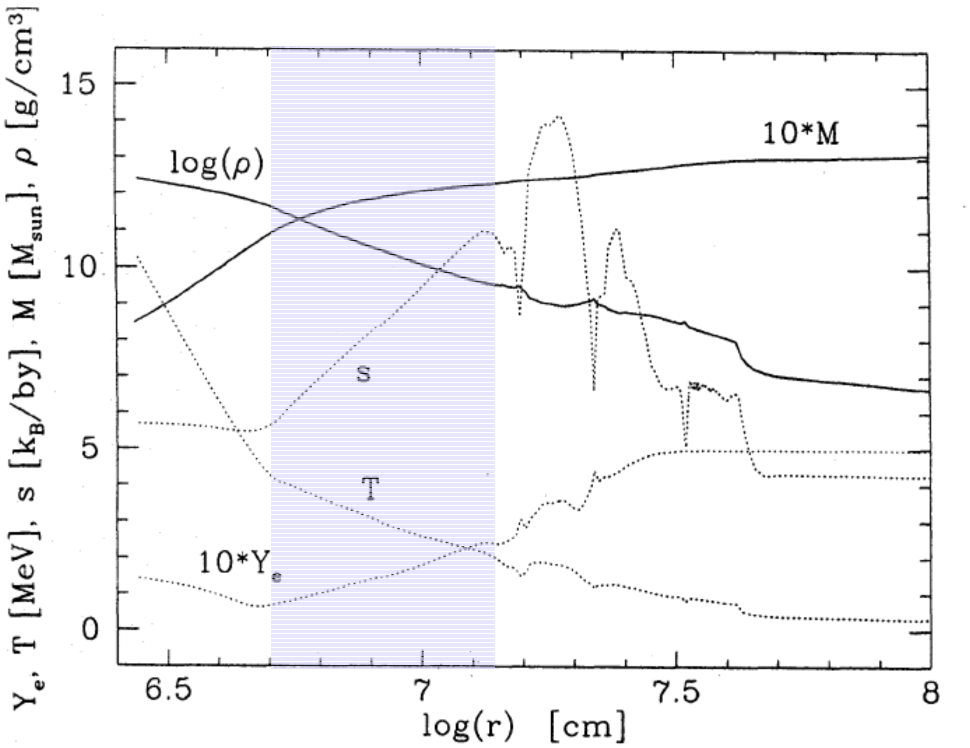
★calculation region : 6.6< logR [km] <7、 9< log ρ [km] <12 (following figure)

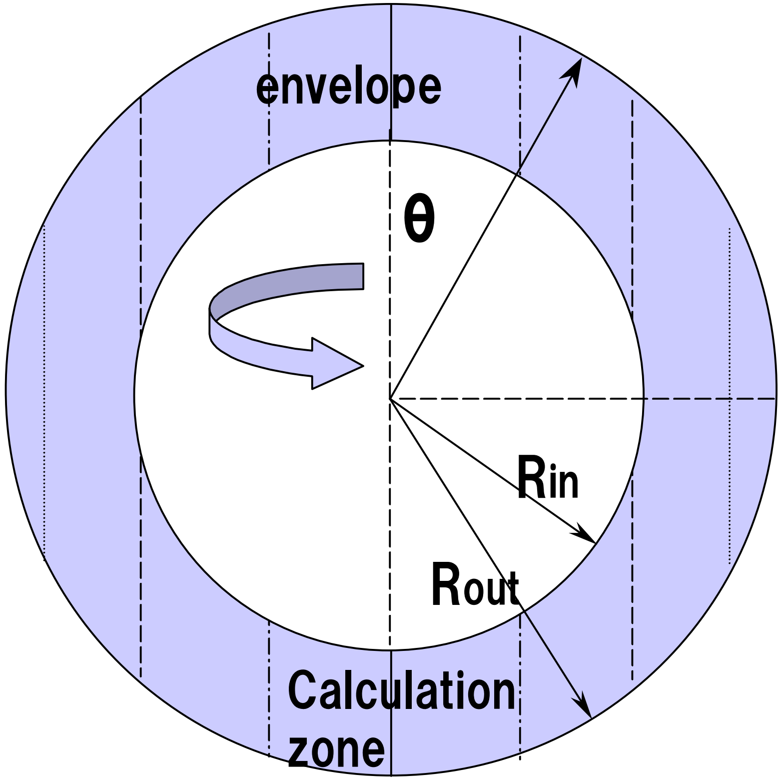
★rotation : power law $\Omega \propto r^{-q}$ ★Heat source at Rin (assuming hot core)

★Boundary condition is fixed at Rout ★Toroidal fields are assumed.

★MHD-Godunov scheme (Sano,Inutsuka&Miyama1998)

★Mesh (r, ϕ ,z) = (64,256,64)





Next challenge – 3D MHD simulation -

Settings of simulation (internal structure : envelope of PNS)

- 3D-MHD global simulation (Cartesian)
- Calculation area (x,y,z) = (30H, 20H, 20H) = (100×100×100)
- Flux tube (or Flux sheet)

◎ Boundary condition

・ fluid velocity	・Magnetic field
Rin : stress free	vacuum condition
Rout: free boundary	vacuum condition

◎ stable stratification

◎ Parker's suggestion

★ Basically, grow at diffusive time

★ interchange mode grows at faster velocity than the thermal diffusion.

★ Parker instability at nonlinear？